Math 180A Homework 5

Winter 2023

Due date: **11:59pm** (Pacific Time) on **Wed. Feb 15** (via Gradescope)

For Homework 5 only, you will receive a bonus point if you collaborate with at least 2 other students in our class!

Section 1 (input directly in Gradescope)

Submit the answers to these problems directly through the Gradescope interface. You do not need to write up or explain your work.

Problem 1 (multiple choice)**.** Which of the following would be modeled best by a Poisson random variable?

(a) The amount of time you have to wait before a car drives by your house.

(b) The number of cars that drive by your house in one hour.

(c) The number of cars that drive by your house before you see a red car.

Problem 2 (multiple choice)**.** X is a random variable, and its cumulative distribution function F_X is

$$F_X(t) = \begin{cases} 0, & t < -3 \\ 0.2, & -3 \leq t < -1 \\ 0.7, & -1 \leq t < 5 \\ 1.0, & t \geq 5. \end{cases}$$

The random variable X is

(a) Discrete

(b) Continuous.

Problem 3 (numerical answer)**.** A random variable X has probability density function

$$f(x) = \begin{cases} cx^3, & 0 \leq x \leq 1 \\ 0, & \text{otherwise}, \end{cases}$$

where c is some positive constant. What is c?

Problem 4 (numerical answer)**.** A random variable X has density function

$$f(x) = \begin{cases} 6x(1-x), & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

Compute $E[X]$.

Section 2 (upload files)

For each problem, write your solution on a page by itself, and upload it as a separate file to Gradescope (either typed or scanned from handwritten work). You should write your solutions to these problems neatly and carefully and provide full justification for your answers.

Problem 5. Let X be a continuous random variable with probability density

$$f_X(x) = \begin{cases} 1/6, & -2 \le x \le 0, \\ 2/9, & 0 < x \le 3, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute $\mathbb{E}[X]$.

(b) Compute $\mathrm{Var}(X)$.

(c) Compute $\mathbb{E}[(X-1)^2]$.

Problem 6. *(This is Exercise 3.37 from Anderson, Seppäläinen, and Valkó's book.)* Suppose that a random variable X has the cumulative distribution function

$$F(x) = \begin{cases} \frac{x}{1+x}, & x \ge 0 \\ 0, & x < 0. \end{cases}$$

(a) Find the probability density function f.

(b) Calculate $P(2 < X < 3)$.

(c) Calculate $E[(1+X)^2 e^{-2X}]$.

Problem 7. You and your friends go to Joshua Tree National Park to watch the peak of an upcoming meteor shower. According to astronomers, the expected number of meteors you will see is 150 per hour. What is the probability that you will see at least two meteors in the first minute? **Hint:** Convert 150 meteors per hour to meteors per minute, then choose an appropriate random variable to model this situation. *(You will have to make some assumptions for this problem, and that's okay!)*

Problem 8. A *standard Cauchy random variable* is a random variable X with the probability density function

$$f(x) = \frac{1}{\pi(1+x^2)}.$$

(a) Verify that f is actually a density function (it is nonnegative and integrates to 1).

(b) Show that $E(|X|) = \infty$. **Hint:** Set up $E(|X|)$ as an integral from $-\infty$ to $+\infty$, and then break it up into two pieces: $-\infty$ to 0 and 0 to $+\infty$.

(c) (Bonus — NOT TO BE TURNED IN) Can you find $E[X]$?

(d) (Bonus — NOT TO BE TURNED IN) Consider a line through the origin in $\mathbb{R}^2$ whose angle Θ is chosen uniformly at random. Let X be the slope of the line. Show that X is a standard Cauchy random variable.

Problem 9. (Bonus — 1 point) Choose an actual data set that you think will be well-modeled by a Poisson distribution, and compare it with the "theoretical data" you would expect if it were indeed generated by a Poisson distribution. Include a clear reference to the data set you used (e.g., a URL).

If you want a concrete suggestion, you can look at the number of goals per game in the English Premier League last season. In this case, if we were to model the number of goals in a given game as a Poisson random variable X, then we could use this assumption to find the expected number of games in which no goals are scored (i.e., where $X = 0$), in which 1 goal is scored (i.e., where $X = 1$), etc. We could then compare this with the actual number of games having each possible score, and see if our choice of a Poisson random variable seems reasonable in light of this comparison.