Math 180A Homework 6

Winter 2023

Due date: **11:59pm** (Pacific Time) on **Wed. Feb 22** (via Gradescope)

Section 1 (input directly in Gradescope)

Submit the answers to these problems directly through the Gradescope interface. You do not need to write up or explain your work.

Problem 1. *(This is Exercise 3.18 from Anderson, Seppäläinen, and Valkó's book.)* Let X be a normal random variable with mean 3 and variance 4.

Note: for this problem, you may find it helpful to use a table of values for $\Phi(x)$.

1. Find the probability $P(2 < X < 6)$. Which of the following is the closest decimal approximation to this probability?

   (a) 0.0228
   (b) 0.6247
   (c) 0.2417
   (d) 0.6453

2. Find the value of c such that $P(X > c) = 0.33$. Which of the following is the closest decimal approximation to this value?

   (a) 3.88
   (b) 4.50
   (c) 2.12
   (d) 0.44

3. (numerical answer) Find $\mathbb{E}(X^2)$.

Problem 2 (multiple choice)**.** Suppose the wait times in an emergency room are exponentially distributed with a mean wait time of 40 minutes. What is the probability that a patient waits more than t minutes, for $t > 0$?

(a) $1 - e^{-t/40}$

(b) $1 - e^{-40t}$

(c) $e^{-t/40}$

(d) e^{-40t}

Section 2 (upload files)

For each problem, write your solution on a page by itself, and upload it as a separate file to Gradescope (either typed or scanned from handwritten work). You should write your solutions to these problems neatly and carefully and provide full justification for your answers.

Problem 3. One day at noon, you have a sudden craving for a California burrito from the Taco Stand, and you decide to take the bus from campus into town. Starting at noon, suppose that the number of minutes until the first bus arrives is an exponential random variable with parameter 1/20.

(a) If you have already been waiting for 10 minutes (i.e. it is 12:10pm) and the first bus has not yet come, what is the probability that you will have to wait at least 15 additional minutes?

(b) Repeat part (a) under the assumption that the number of minutes from noon until the first bus is uniformly distributed over [0,40].

(c) Explain how the answers to (a) and (b) relate to whether exponential and uniform random variables are memoryless.

Problem 4. *(This is Exercise 3.67 from Anderson, Seppäläinen, and Valkó's book.)* Let $Z \sim \mathcal{N}(0, 1)$ and $X \sim \mathcal{N}(\mu, \sigma^2)$. This means that Z is a standard normal random variable with mean 0 and variance 1, while X is a normal random variable with mean μ and variance σ^2.

(a) Calculate $E(Z^3)$ (this is the *third moment* of Z).

(b) Calculate $E(X^3)$.
Hint. Do not integrate the density function of X unless you love messy integration. Instead use the fact that X can be represented as $X = \sigma Z + \mu$ and expand the cube inside the expectation.

Problem 5. *(This is Exercise 4.48 from Anderson, Seppäläinen, and Valkó's book.)* Let $X \sim \text{Exp}(2)$. Find a real number $a \in [0, 1]$ such that the events $\{X \in [0, 1]\}$ and $\{X \in [a, 2]\}$ are independent.

Problem 6. You own a store that sells a stove for \$1000. You purchase the stoves from a distributor for \$800. The lifetime of the stove is described by an exponential distribution with parameter $\lambda = 1/10$.

You would like to offer an extended warranty on the stove: The customer will pay you \$C for the warranty. If the stove breaks within r years, then you will replace the stove (which will cost you \$800). If the stove lasts longer than r years, you pay nothing to the customer.

(a) What is your expected profit from the warranty? (This answer will depend on C and r.)

(b) If you want the warranty to last five years, what will the cost C of the warranty need to be for you to break even (profit $= 0$) on average?

Problem 7. Suppose X, Y have joint density function

$$f(x, y) = \begin{cases} c(xy + y^2), & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

for some constant c. Find the value of c.