1. Let $X$ be a path connected topological space which is NOT compact. Show that $H^0_{\text{comp}}(X) = 0$.

2. Compute $\text{Ext}_{\mathbb{Z}}^1(\mathbb{Z}/n, \mathbb{Q}/\mathbb{Z})$.

3. Compute the cohomology ring of $\mathbb{C}P^3 \times \mathbb{R}P^2$.

4. Let $X$ be the suspension of $\mathbb{R}P^2$, i.e.,

$$X = \mathbb{R}P^2 \times [0, 1]/(\mathbb{R}P^2 \times \{0\}, (\mathbb{R}P^2 \times \{1\})$$

Prove that $X$ is not homotopy equivalent to a compact manifold.

For the last two problems we'll need the following definition: If $M, N$ are connected closed oriented manifolds of the same dimension and $f : M \to N$ is a continuous map, then the degree $\deg(f) \in \mathbb{Z}$ is defined by

$$f_*[M] = \deg(f) \cdot [N] \in H_n(N; \mathbb{Z}).$$

5. Prove that any continuous map $f : S^2 \times S^2 \to \mathbb{C}P^2$ has even degree.

6. Let $M^n$ be a connected closed oriented manifold and assume that there is a degree one map $f : S^n \to M^n$. Show that $H_i(M; F) = 0$ for all $0 < i < n$ and any field $F$. 