Optimal hybrid control of a compressor field

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Abstract
The optimal control of a compressor field is discussed. The plant is modelled as a hybrid system. A dynamic programming approach as suggested by Branicky is taken. In order to obtain easily applicable control laws, two mappings are proposed for systems with only one optimal trajectory through each point in the hybrid state space.

A receding horizon control provides a smooth and simple control law and can guarantee stability to a certain point, but does not precisely attain the target state due to a compromise between cost and target. A control law obtained from the hull of the optimal reachability set shows time optimal performance and a certain robustness, but does not maintain the target state at optimal cost.

Robustness of the controllers with respect to changes in the gas flow demand is discussed in an example.

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1 Introduction
Systems with event-driven as well as time-driven components, also referred to as hybrid systems, have received increasing interest over the past years. The control of such a system is a particularly difficult task if switchings and jumps occur that completely change its behaviour. A scheme for the optimal control of hybrid systems using dynamic programming has been proposed by Branicky [2]. In this paper, this scheme is applied to control a gas compressor field minimising running and switching cost. The model chosen is stable and of low dynamic order, but highly non-linear and of sufficient complexity to be interesting. Its event-driven component consists of starting up or shutting down the individual compressors. The dynamic optimisation problem is solved using a dynamic programming strategy based on the Generalised Value Iteration proposed by Branicky [2]. The solution strategy is, in brief, to minimise a cost function by finding out what is the best action to take, for every point in the state space and at every time within a set time horizon. We consider a finite horizon problem as the equality constraints are not met precisely if the optimum is found by minimising a single overall cost function. Terminal constraints are therefore added, which can be met easier by standard algorithms.

Solving the dynamic programming problem is computationally intensive, and its solution requires a large amount of resources. This prevents its application as an online controller. To overcome this obstacle, the calculation is carried out offline and simpler sub-optimal maps are then used as control laws for online control. Two such control laws are proposed in this paper. The first one is obtained by sampling the dynamic programming solution. The samples are then applied as a continuous control law. This corresponds to a receding horizon or model predictive control. It is shown that the control scheme is to a certain point asymptotically stable but compromises between minimising cost and attaining the target state. The second control law is obtained by considering the boundary of the backwards reachability set and corresponds to time optimal or bang-bang control. The robustness of the suggested controllers is addressed with a simulation of a drop in demand mass flow. Finally, switching delays are discussed and suggestions are made for their inclusion in the optimisation.

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2 Modelling

2.1 Individual gas compressors

The gas compressor model is taken from Paice [8]. It is a simple model which assumes: static compressor characteristic, low-side compressor conditions independent of downstream system, negligible temperature dynamics, nonnegative mass flows.

The system is then characterised by equations for the static characteristics, with \( m_1 = g(u_1, p_2) \), and the temperature change across the compressor and the valve model being taken from the literature, and the following state equation:

\[
\dot{p}_2 = \frac{R}{V}(m_1 - m_2 - m_3) \cdot T_2
\]

Where
- \( m_1 \): gas compressor mass flow
- \( m_2 \): output demand
- \( m_3 \): recirculation valve mass flow
- \( p_1 \): inlet pressure
- \( p_2 \): discharge pressure
- \( R \): gas constant
- \( V \): volume of effective output piping
- \( g(\cdot) \): static compressor characteristic
- \( T_1 \): inlet temperature
- \( T_2 \): output temperature
- \( u_1 \): inlet vane angle
- \( u_2 \): recirculation valve stroke

From the assumptions, \( p_1, T_1 \) and \( m_2 \) are considered boundary conditions and \( m_1, T_2, \) and \( m_3 \) can be seen as intermediate variables.

2.2 Compressor field

The gas compressor field is assumed to be consist of several single stage compressors connected in parallel. In order to obtain a simple and transparent model, the following simplifications are made: There is a single recirculation valve, the output pipe junction uniform temperature and simplifications are made: There is a single recirculation to obtain a simple and transparent model, the following state equation, as well as event-driven dynamics, the discontinuous gas flow through the RCV, i.e. minimise \( u_{RCV} \), while observing the constraints to avoid surge \( (\phi(p_1, T_1, p_{out}, m_k) \geq c_k) \) and observe control saturation: \( u_k \in [0,1] \).

These criteria are now extended to consider the costs of operating the plant. To do so, we define
  - running costs \( k_i(m_i) \) for each compressor and
  - switching costs \( K(x_{D,1}, x_{D,2}) \) from discrete state \( x_{D,1} \) to \( x_{D,2} \) are assigned.

The minimisation of RCV gas flow is then implicitly taken care of, since the mass flow through the compressors incurs costs while the RCV is open. This is then formulated as an optimization problem. as follows:

P1: minimize \( f(p_1) \)

\[ \sum_{k \in [0,1]} u_k \in [0,1] \]


2.3 Hybrid model of the compressor field

To account for both the dynamic behaviour and the switching between compressors, the plant can be modelled as a hybrid system.

A hybrid system is characterised by the interaction of time-driven and event-driven systems and their different interactions [7]. The time-driven dynamics, often referred to as continuous dynamics, are usually described with a set of differential or difference equations. The event-driven or discrete dynamics can be modelled as automata [2] or more efficiently as interpreted Petri nets (IPN) as suggested by Nenninger [6].

A hybrid system can be interpreted as a generalised dynamic system with discrete and continuous states [2]. Each discrete state can be associated with a continuous dynamic system. The discrete dynamics then switch between those systems either autonomously, i.e. when the state enters an autonomous jump set, or, controlled, i.e. when the state is inside a controlled jump set. In this paper we take a slightly different approach. According to [7] the continuous dynamics are modelled as a single set of state equations dependent on the discrete dynamics via an interface vector \( \nu(x) \). The discrete part of the compressor field describes the state of the compressors as on or off. The discrete controls are identical to the discrete states. The extended state space model includes multipliers that are zero if the compressor is off.

3 Hybrid Dynamic Optimisation

The plant integrates time-driven dynamics, the continuous state equation, as well as event-driven dynamics, the switching of compressors. It is therefore modelled as a hybrid system.

3.1 Control and optimisation problem formulation

As in [8], the basic control objectives for a gas compressor are to maintain a constant discharge pressure, \( p_{out} = p_{goal} \), while minimising the gas flow through the RCV, i.e. minimise \( u_{RCV} \), while observing the constraints to avoid surge \( (\phi(p_1, T_1, p_{out}, m_k) \geq c_k) \) and observe control saturation: \( u_k \in [0,1] \).

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P1: minimize \( f(p_1) \)

\[ \sum_{k \in [0,1]} u_k \in [0,1] \]
where
\[ V(p_0) = \int_0^T \sum_{i=1}^{n} k_i (\dot{m}_i(t)) \cdot v_i(t) \, dt + \sum_{i=0}^{n_{SW}} K(x_{\sigma_i}, x_{\sigma_i}) \]
\[ + k_p \int_0^T (p_{out}(t) - p_{goal})^2 \, dt \]
subject to
\[ C1: \quad \dot{p}_{out} = \frac{R}{V} \left( \sum_{k=1}^{n} g_k(u_{i,k}, p_{out}) \cdot \sum_{i,k} - \dot{m}_{\text{RCV}} - \dot{m}_{\text{out}} \right) \cdot T_{out}, \]
\[ p_{out}(t = 0) = p_0, \]
\[ C2: \quad \phi(p_{i1}, T_{i1}, p_{out}, \dot{m}_i) \geq -c_i, \]
\[ \dot{m}_i(t) = g_{i}(u_{i,j}, p_{out}), \quad \forall i \in 1,...,n \]
where
\[ T \quad \text{time horizon}, \]
\[ n_{SW} \quad \text{number of switchings}, \]
\[ \sigma_i \quad \text{time at switching } \#i. \]

### 3.2 Dynamic Programming

Following Nellinger, we restrict the possible switching times to occurring regularly every \( \delta \) seconds. The problem is thus reformulated as the combination of a discrete and a continuous optimisation problems, with the value function at the time steps being given by the minimum of the solution of these two functions.

When considering this problem as an infinite horizon optimisation problem, this presents a simple strategy for finding a cost optimal control law. However, from a practical point of view, there are two crucial drawbacks. Firstly, the numerical optimisation of penalty functions requires sophisticated methods, mostly due to the difference in scale between the cost of the penalty and the running cost. Secondly, the main objective of the control problem is to reach the target state, \( p_{\text{goal}} \).

To avoid these problems, the goal of attaining the target state was added as a terminal constraint to the continuous sub-problem. With this addition, the penalty cost for not attaining the target state can be dropped for small time steps, since it has negligible effect on the results. The time step \( \delta \) then becomes the determining factor in the trade-off between reaching the goal state and minimising the cost. As \( \delta \to 0 \), the continuous control law becomes time optimal while the choice of switching only considers the cheapest route.

In defining the constraints it is important to put the same criteria on the discrete and continuous parts of the optimisation problem. Therefore, the new terminal constraint has to be taken into account. A simple way to do so is to add a penalty cost when evaluating the discrete controls. An even simpler suboptimal way is to add the penalty cost only at the time horizon \( T \).

#### 1.1.1 Numerical solution

In this first approach, switching costs were not taken into account, while the surge constraint was considered with an exponential penalty function. The numerical solution was found using the gradient method algorithm “FOPC” from [4] which was adapted to handle control saturation. The cost function as well as the controls were approximated by linear interpolation between the discretized values.

#### 1.1.2 Example

To illustrate the algorithm, consider two compressors: one cheap, the other expensive as in Fig. 2.

Three discrete states are considered:
1. cheap compressor on,
2. expensive compressor on,
3. both compressors on

The time horizon is 1 s, the sampling time 0.2 s. Fig. 3 shows the optimal trajectories over varying initial pressure. Two sample trajectories are drawn bold. Starting at a higher pressure, the pressure is lowered with compressor 1 running at minimum speed, the RCV fully open. When starting at the lower pressure, the pressure is raised with compressor 1 at full power for 0.4 s. Then, it is switched to state 3, both compressors running. At 0.8 s, the more expensive compressor 2 is switched off and the target state attained at horizon time 1 s.

![Figure 2: Running cost of example compressors](image-url)
are calculated up to the time horizon. The first sample is applied. For the next sample, new controls for several samples are calculated, but only the initial controls for the current sample are applied continuously. As the sampling time \( \delta \) approaches 0, the continuous control is approached.

If the time horizon is a single sampling time, the control law is time independent. This is in fact an offline model predictive control. There, the controls for several samples are calculated, but only the first sample is applied. For the next sample, new controls are calculated up to the time horizon.

Here, the law is applied continuously. The justification for this approach is that for each new state \( x \) which is reached, \( u(x,t) \) is indeed the optimal control for a (infinitely) small time. Note that it is an optimal control relative to the time horizon \( T \). After that small time, a new optimal control at time \( t + \varepsilon \) has to be applied, but since the state has moved on as well, a control \( u(x + \varepsilon, t + \varepsilon) \) is needed. Now, for a time-invariant system, this is the same as the control \( u(x + \varepsilon, \tau) \) for the problem treated at the time \( \tau = t + \varepsilon \) with time horizon \( T + \varepsilon \).

In other words, when applying the sampled control \( u(x,t) \) continuously, the plant is controlled with respect to a receding horizon, so as to meet the terminal constraints at this receding horizon. At the next sampling time \( t + \delta \), the next control \( u(x,t + \delta) \) is used.

### 1.3.1 Stability

If the cost incurred by applying the control law is always decreasing it can be followed that the state converges and the system is asymptotically stable. This shall be examined for the receding horizon control above. Following a proof by Bemporad [1] about discrete-time receding horizon control and adapting to a continuous system, it is possible to show that if the time horizon is chosen sufficiently small, convergence is guaranteed outside a neighbourhood about the set point. Introducing a terminal constraint as in DP1, does not directly improve the receding horizon controls. The shorter the time horizon the closer the state converges towards the target state. However, the target state is never met precisely.

To avoid this problem, controls for different time horizons can be combined to obtain the desired characteristics. Alternately, as is often done in the Model Predictive Control literature, a local asymptotically stable control law may be applied instead of the optimal control law once the state reaches a neighbourhood of the target state.

### 1.3.2 Simulation Results

Various sample trajectories of the receding horizon control are shown. The initial pressure was chosen to take on various values above and below the pressure set point. Simulations show that the trajectories converge smoothly towards the set point. Fig. 4 shows the corresponding controls. Note that for \( u=0 \) the output of a compressor is greater than zero unless it is switched off.

### 1.4 Mapping via the hull of the reachability set

Given that only one optimal trajectory passes through each point in the state-space, a simplified control law can be obtained from the hull of the backwards reachability set. Tracing back from the target state at time \( T \), the union of all points of the state space on all optimal trajectories until time \( t \) shall be called the optimal reachability set (ORS).
By definition, its hull contains the maxima and minima of the ORS at time $t$ and converges to the target state at time $T$. If this set is continuous, this entails that the hull contains in fact all points of the ORS at some time. If that is the case, the hull of the ORS maps a control to every point inside. This can be stored as a time-independent control law, the dimension of which is decreased by 2, compared with the laws for every point in space and time. The resulting control law is optimal for the remaining time since for every point on an optimal trajectory the remaining trajectory is optimal.

A drawback of this method is that it does not maintain the target state with minimum cost after it has been reached. This follows from the definition, since the criteria was to reach the target state, not to hold it. Comparing the cost from the following example with the receding horizon control, it can be observed that the running cost after the target state has been reached is significantly higher. Figure 7 shows sample trajectories with the controls from the hull of the ORS. The costs are notably higher than with the moving horizon control.

### 1.5 Controller Performance

Before the suggested control laws can be applied to the plant, it has to be verified that the controller handles uncertainties or changes in the parameters and boundary conditions. A complete analysis of this problem is beyond the scope of this paper, so the topic is addressed with the following example. During the operation of the compressor, the demand in mass flow can suddenly drop to a percentage of the normal level. This has been simulated with both of the suggested control laws.
As fig. 7 shows, both handle the disturbance quite well. Note that the scaling of the pressure graph of the reachability control is significantly larger and the pressure practically does not vary at all. However, the controls chatter at the changing mass flow.

4 Conclusions

The compressor field has been modelled as a hybrid system and running and switching costs have been assigned. The optimisation was carried out using a dynamic programming approach suggested by Branicky. In order to obtain easily applicable control laws, two mapping strategies have been proposed for systems with only one optimal trajectory through each point in the hybrid state space.

A receding horizon control provides a smooth and simple control law and can guarantee stability to a certain point, but does not precisely attain the target state due to a compromise between cost and target. A control law obtained from the hull of the optimal reachability set shows time optimal performance and a certain robustness, but does not maintain the target state at optimal cost. Both controllers showed robust performance when load variations were simulated.

Future research should provide more efficient ways to obtain simple control laws, e.g. by time-discretisation of the plant model. The question of robustness has been raised and should receive further attention.

The computing power of today’s personal computers provides a way to obtain control laws by simple and reliable numerical methods without a manpower-consuming in-depth system analysis. The computed controller can then be tested with a more accurate model of the plant. As simple maps of states to controls, the control laws can be stored and applied using a low-cost controller since they only require memory, not computing power.

References


