Path Following Regulator for Neural Network Implemented Control System of Adaptive Mobile Robot Moving with a Set Speed

Viacheslav Kh. Pshikhopov*, Yuriy V. Chernukhin**
*Department of Automatic Control Systems, 
**Department of Computing Sciences
Taganrog State University of Radio Engineering, 
Nekrasovsky str., 44, GSP-17A, Taganrog, 347928, Russia
pshichop@pbox.ttn.ru

Keywords: mobile robot, control, neural network.

Abstract

In this work a synthesis procedure of path following regulator for network control system of adaptive mobile robot is presented. The robot’s motion trajectories are set as quadratic forms of space coordinates. The synergetic path following regulator ensures asymptotic stability of the planned trajectories, accounts for the non-holonomous qualities of the control object model and maintains a constant set motion speed along a desired path. Synthesis example, calculation algorithms of the quadratic forms coefficients and modeling results are adduced.

1 Introduction

One of the main problems in organization of intellectual behavior of adaptive mobile robots (AMR) functioning in a complex non-formalized environment is the problem of automatic path routing and its effective performance. The work [1] shows bionic method of solution of this problem basing on neural network implemented control systems. The idea of this method is that in the process of interaction with a non-formalized environment the plan of the environment is reconstructed by the robot’s control system using the sensor system. This is done each time the robot performs any elementary operation by its actuators. This plan reflects the current location of the target, obstacles, the robot itself and of the areas free for motion rated by their difficulty. This plan is reflected in the states of neural elements in the physically realized onboard planning neural network. Then the system finds a gradient of the functional determined by the multitude of the possible trajectories reaching the goal. After that the neural decision making system forms a vector of elementary motion and the effectors implement it. This vector is directed along the anti gradient of functional found on the environment plan obtained in the neural networking control system. These actions are repeated until the goal is reached.

As we see from the description above, according to the bionic method organization of the purposeful AMR motion comes down to real-time solution of the following three tasks:
- adequate perception of the information about the environment and forming the current map (plan) of motion through it;
- neural network planning of the optimal robot's motion trajectory to the goal on the plan and making the decision about the direction of motion in the current situation;
- effective performing of the planned actions in the real environment.

Theoretical and experimental researches of this method have shown its high effectiveness in solution of a number of applied tasks of adaptive robotics [2]. However this research revealed its drawbacks connected to the fact that hardware expenditures on realization of the neural network grow fast together with the increase of the dimension of the real functioning environment. There were other difficulties in performing the planned trajectories.

To overcome these drawbacks an approach was proposed in the work [3]. It is an approach of hierarchical planning of the robot's motion based on consecutive usage of the neural network equipment for large-scale step-by-step planning, which is performed starting from the least accurate top strategic level and going down to the most accurate local tactical level. This approach allowed to increase effectiveness of neural network planning but didn't solve the problem of increasing the effectiveness of planned trajectories performing.

Rather interesting results were achieved in the works [4,5,6]. So in the work [4] the authors using the solution of inverse kinematics synthesized control guarantying asymptotic stability in the large of the parametrically set trajectories. The work does not consider the trajectoryal tasks in the closed-loop form. The work [5] presents control algorithms for non-holonomous systems. These controls ensure the set qualitative qualities of the trajectoryal manifolds but the problem of forming the set contour speed still remains. Rather original control procedures for mobile robots ensuring exponential stability
of the set trajectories are presented in the work [6]. However the presented approach assumes presence of devices approximating the desired trajectory by the arcs and straight lines which lowers the robot's functional capabilities. Besides, authors didn't account dynamic qualities of the control objects and this requires additional research of the qualitative qualities of the closed loop systems.

This work proposes an approach based on joining the bottom tactical planning level with the level of performing the planned trajectories and their realization not by the neural systems but by the specially synthesized path following regulators. The proposed approach allows to increase the accuracy of the planned trajectories performing by the effector system of the AMR. So the robot precisely follows the routs planned on the neural network strategic level and allows to increase the effectiveness of not only planning but also performing the planned actions.

Contour regulator forms the local motion routs and, firstly, the trajectories of obstacles rounding at an allowed constant speed. These trajectories are formed as a result of approximating the obstacles contours by the quadratic curves. Routes of the necessary motions are read from the output of the neural network decision making system. When needed the obstacles contours are read from the environment passability plan in the form of binary images. These contours, in turn, are approximated by the second order curves and are presented in the form of performed trajectories. By means of synthesized controls these trajectories are transformed into attracting manifolds - attractors being the asymptotically stable solutions of the closed-loop systems. Unlike the known methods the proposed approach allows to join the neural network modules of AMR motion planning with the modules of direct control of its drives.

2 Task Statement and Control Synthesis

The task of AMR local trajectories planning and control of their realization can be formulated in the following way. Assume that kinematical scheme of the mobile robot has one controlled front wheel and two driving rear wheels. The scheme is presented in fig. 1 [7]. Assume that the rear wheels drives are controlled by one channel and the drive of the front wheel by the other one. Then the mathematical model adequately reflecting the AMR dynamics has the following form:

$$\dot{x} = F(x) + B(x) \cdot U,$$  \tag{1}

where $x$ - $n$ dimensional vector of controllable and observable coordinates; $F$ - $n$ dimensional vector of nonlinear elements; $B$ - $(n \times n)$ matrix of coefficients; $U$ - $n$ dimensional vector of controlling torques; here $n = 2$.

The controlled coordinates are angular velocity of the rear wheels $\omega$ and rotation angle of the front wheel $\beta$.

Assume that we also know the kinematical limitations determined by AMR layout scheme that have the following form:

$$\Sigma = \Psi(p, \dot{p}, c) = 0,$$  \tag{3}

where $\Sigma$ - differentiable vector-function of its arguments, $\dim \Sigma = n$; $c$ - some constants.

Manifold (3) is the intersection line of the manifolds $\Sigma_1(p, c)$ and $\Sigma_3(p, c)$ in the space $R^m$. These submanifolds reflect the requirements to the space coordinates and speeds. Now let's formulate the general synthesis task: it is necessary to synthesized such a control $U(x)$, that would ensure asymptotic stability (in the large) of the trajectories (3) transforming them into the attractors.

Solution of such a task for manipulating robots was discussed in the works [8,9]. Now let's move to solution of the AMR (1), (2) contour control task inside the set task.

Let's note that all the multitude of AMR trajectories determined by technological task can be described rather fully by quadratic forms of state coordinates $p_1, p_2$ in the form of the following equation:

$$a_1p_1^2 + a_2p_2^2 + a_3p_1 + a_4p_2 + a_5 = 0,$$

or in the matrix form

$$\Sigma_i = p^T N_i p' + N_j p + N_k = 0,$$  \tag{4}

where $p' = [p_3, p_2]^T$, $N_i = \begin{bmatrix} 0 & a_3 & a_4 \\ a_3 & 0 & a_5 \\ a_4 & a_5 & 0 \end{bmatrix}$.
Depending on the matrices \( N_i, i = 1, 3, \) equations of the form (4) can describe circumferences, ellipses, straight lines etc.

Let's set s requirement to the contour speed of motion along the trajectory (4) in the form of the following equation:

\[
\Sigma_a = p^T p^* - V_k^2 = 0,
\]

where \( V_k \) – contour speed determined by the task statement. The AMR motion speed with respect to the trajectory (5) we set by the following equality:

\[
\dot{\Sigma}_t = 2p^T N_1 p^* + N_2 p^* = 0.
\]

Obviously, value of the contour speed \( V_k \) should satisfy the inequality

\[
V_k \leq V_{\text{max}},
\]

where \( V_{\text{max}} \) – maximal contour speed determined by the power AMR capabilities.

Then submanifolds \( \Sigma_t \) and \( \Sigma_s \) can be formed as vectors

\[
\Sigma_t = \begin{bmatrix} \Sigma_1 \\ 0 \end{bmatrix} = 0,
\]

\[
\Sigma_s = \begin{bmatrix} \Sigma_1 \\ p^T p^* - V_k^2 \end{bmatrix} = 0,
\]

And manifold \( \psi \) (3) can be described by their linear combination

\[
\psi = \Sigma_t + \alpha \Sigma_s = \begin{bmatrix} \Sigma_1 \\ 0 \end{bmatrix} + \alpha \begin{bmatrix} \Sigma_1 \\ p^T p^* - V_k^2 \end{bmatrix} = 0, \\
\alpha = (0, -V_k^2),
\]

where \( \alpha \) – positively determined matrix of set constant coefficients, \( \dim A = nxn \); \( 0_0 \) – vector of zero elements; \( T \) – transposing symbol.

Manifold (9) sets the desired AMR motion trajectory in the space \( R^{2m} \) of phase coordinates.

In order to transform the trajectory (9) into an attractor it is necessary to require this trajectory to be the solution of the differential equation of the following form [8]:

\[
T \dot{\psi} + Q(\psi) = 0,
\]

where \( T \) – positively determined matrix of constant coefficients, \( \dim T = n \times n \), \( n = 2 \); \( Q \) – some vector function of macrovariable \( \psi \).

Assuming \( Q = \psi \) and substituting expression (9) into the Eq. (10) and accounting the object model (1) and (2) we get:

\[
U(x) = -[k_1 R B]^{-1} [k_1 (R F + L M_2) + k_2 p^* + k_3].
\]

\[
k_1 = TA \left[ \begin{array}{c} D \\ 2p^T \end{array} \right], \quad R = \left[ \begin{array}{cc} \frac{\partial M_1}{\partial \alpha} & \frac{\partial M_1}{\partial \beta} \\ \frac{\partial M_2}{\partial \alpha} & \frac{\partial M_2}{\partial \beta} \end{array} \right], \quad L = \frac{\partial M_2}{\partial \alpha}.
\]

\[
k_2 = A \left[ \begin{array}{c} D \\ p^T \end{array} \right] + TA \left[ \begin{array}{c} 2p^T \end{array} \right], \quad k_3 = \Sigma_s + AV.
\]

here \( D = 2p^T N_1 + N_2 \).

It should be mentioned once again that in order to realize algorithm (11) we need sensor support of the observation process for coordinates \( x \) and coordinates \( P \).

3 Example

Now let's consider the path following regulator (11) synthesis procedure satisfying the set task for the mobile robot described by the following model [10]:

where \( s() = \sin() \), \( c() = \cos() \); \( r \) – wheel radius, \( , \)

\( r = 0,2 \) m; \( h \) – distance from the rear wheels axis to the foot point, \( h = 0,5 \) m; \( d \) – distance from the rear wheels axis to the front wheel axis, \( d = 0,8 \) m; \( a_1 = 1/0,7, b_1 = 1/0,7, b_2 = 5 \) – parameters of the robot's drives.

Assume that it is necessary to organize asymptotically stable motion of the robot (12) along the quadratic form (4) with a contour speed \( V_k \).

According to the algorithm (11) we get:

\[
k_1 = \begin{bmatrix} 2a_1 p_1 + a_1 & 2a_1 p_2 + a_1 \\ 2p_1 & 2p_2 \end{bmatrix},
\]

\[
k_2 = \begin{bmatrix} b_1 (2p_1 + sp_1) + a_1 & a_1 (2p_1 + sp_2) + a_1 \\ p_1 & p_2 \end{bmatrix},
\]

\[
k_3 = \begin{bmatrix} a_1 p_1^2 + a_2 p_2^2 + a_1 p_1 + a_2 p_2 + a_1 \\ -sV_k^2 \end{bmatrix},
\]

\[
R = \begin{bmatrix} \cos -\beta - \rho \beta \cos h d & \rho \beta - \rho \beta \sin h d & b \beta \cos \delta \sin h d \\ \rho \beta - \rho \beta \cos h d & \cos -\beta - \rho \beta \cos h d & b \beta \sin \delta \cos h d \\ \rho \beta - \rho \beta \sin h d & b \beta \cos \delta \cos h d & \cos -\beta - \rho \beta \sin h d \end{bmatrix},
\]

\[
L = \begin{bmatrix} \rho \beta - \rho \beta \cos h d & b \beta \sin \delta \cos h d \\ b \beta \cos \delta \sin h d & \rho \beta - \rho \beta \cos h d \end{bmatrix}.
\]
Consider the procedure of determining the quadratic forms (4) coefficients as a function of the obstacles digital image read from the environment passability plan (fig. 2), acquired as a result of preliminary processing of the information from the environment sensors (system of technical vision, laser distance gauge etc.). In future we'll assume that image is binary, i.e. intensity levels take the values of 0 and 1. Assume that as a result of performing the image describing procedures the obstacles boundaries (contours) were found in the form of discrete arrays \( D = \{k,(i,j)\}, \ell = 1,LL, \) where \( LL \) — number of points constituting the contour; \( i,j \) — abscissa and ordinate of the \( \ell \)-th point. The mentioned procedures are well known and are widely used in the systems of technical vision \([11,12,13]\).

In order to determine the quadratic form coefficients, describing circumference, it is necessary to determine the mass center \( (x_0, y_0) \) coordinates according to the expression \([13]\):

\[
x_0 = \frac{1}{LL} \sum_{i=1}^{LL} x_i, \quad y_0 = \frac{1}{LL} \sum_{j=1}^{LL} y_j.
\] (14)

Let's introduce the parameter \( R^2 \) determined as

\[
R^2 = \max \left\{ (x_i - x_0)^2 + (y_i - y_0)^2 \right\}, \ell = 1,LL.
\] (15)

As a result of rather simple transformations we get the coefficients values of the quadratic form (5) describing the circle:

\[
a_1 = 1, \quad a_2 = 1, \quad a_3 = -2x_0, \\
a_4 = -2y_0, \quad a_5 = x_0^2 + y_0^2 - R^2.
\] (16)

For the array presented in fig. 2 we get:

\[
L = 18, \quad x_0 = 9, \quad y_0 = 6, \quad R^2 = 20; \\
a_1 = a_2 = 1, \quad a_3 = -18, \quad a_4 = -12, \quad a_5 = 97.
\] (17)

The circle determined by coefficients (17) approximates a contour of a flat image of an obstacle presented in fig. 2.

Another images approximation method uses polygons \([11]\) and other convex figures.

In case when the desired direction of robot's motion is determined by the angle \( \Delta \Theta \), \( \Delta \Theta = \Delta \Theta_1 \leq \Delta \Theta \leq \Delta \Theta_2 \), which value is formed by the neural system in the corresponding cells of the register \( P \) \([1]\) (see fig.3), then in the Cartesian coordinates \( P_1OP_2 \) tied to the robot's shell the straight line equation has the following form

\[
p_2 = \tan \Delta \Theta_1 p_1,
\] (18)

or in the matrix form:

\[
(-\tan \Delta \Theta_1) \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = 0.
\] (19)

Considering the straight line (19) equation in the system of base coordinates \( P_1OP_2 \), let's assume that robots center of mass has the coordinates \( p_{1C} \) and \( p_{2C} \), and the angle of the coordinate system's rotation is equal to \( \alpha \) according to fig.1. Then the straight line equation in the base system of coordinates is determined by the expression \([14]\):

\[
(-\tan \Delta \Theta \alpha) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} - \begin{bmatrix} p_{1C} \\ p_{2C} \end{bmatrix} = 0.
\] (20)

Respectively, coefficients of the quadratic form (4) take the following values:

\[
a_1 = 0, \quad a_2 = 0, \quad a_3 = -\tan \Delta \Theta_1 \cos \alpha + \sin \alpha, \\
a_4 = \tan \Delta \Theta_1 \sin \alpha + \cos \alpha, \quad a_5 = -p_{2C} + \tan \Delta \Theta_1 p_{1C}.
\] (21)

The AMR (12) behavior modeling results looped by control laws (12), (13) are presented in Figures 4 and 5, where the point \((9.1; 7.5)\) — a starting point for the rounding of same obstacles. AMR contour speed during the motion along the circumference is \( 10 \) m/s, and along the straight line \( 5 \) m/s.

**4 Conclusion**

So in this work we present a procedure of planning the trajectories in the space of AMR phase coordinates as quadratic forms of these coordinates in order to organized mobile robot's control on the tactical level of planning and performing its motion routs in a non-formalized eternal
environment. The synthesized path following regulator ensures asymptotic stability (in the large) of the planned trajectories and accounts the qualities of non-holonomous control object model. The results of computer modeling confirmed the effectiveness of the proposed approach of joining the tactical level of planning the actions of the mobile robot with the level of

there performance with a help of path following regulator. Limitations for the control and phase coordinates were not accounted.

### Acknowledgements

The authors express gratitude to the Russian Foundation for Basic Researches (grant № 99 01-00071) and to the Ministry of Education of Russia (Scientific-Technical Program "Scientific Researches of Universities for Industrial Technologies") for the financial support.

### References


