New Approach to the Design of the Near Time Optimal Path Following Controller for the Manipulating Robots

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Abstract

In presented work, the new approach to the design of the near time optimal path following controller for manipulating robots (MR) is considered. We propose analytical algorithms forming procedure of controls, which account nonlinear, dynamical MR’s qualities and desired motion pattern. The proposed control algorithm and its structural realization allow to organize motion of MR’s effector along the set trajectories with a near maximal contour (trajectory) speed.

1 Introduction

The increasing necessity to intensify the working modes of manipulating robots (MR) creates more and more strict requirements to their control systems. The circle of tasks for which it is necessary to work in the modes ultimate in the sense of speed is rather wide. Most of all it concerns the modes of MR functioning in extremal conditions: in case of increased requirements to performance; in case of joint work with other high productivity robots and devices etc. In each of these cases besides the optimization of speed criterion we state the task of following a set trajectory that can be set as a function of external (working) coordinates or in the space of generalized coordinates. The last condition requires analytical or numeric solution of the inverse kinematics. This is connected to the known difficulties and problems [1]. The equally important factor is the necessity to account nonlinear qualities of MR’s dynamics that are the most characteristic for such modes [2]. Besides, we should ensure asymptotic at all of the planned trajectories. All of this determines actuality of the set task.

Questions of analytical synthesis of controls ensuring asymptotic stability (in the large) of the desired motion trajectories set in the class of quadratic forms were already discussed in the works [3,4,5]. The algorithms proposed there do not require solution of inverse kinematics. There is also no need for presence of interpolators or approximation devices. A constant contour speed was considered in these works and besides these algorithms allow to use the maximum speed curves [6,7]. It should be mentioned that in the work [7] the maximum speed curve (MSC) was used only in analysis of the structure of time optimal robots control while in the work [6] MSC was used for control too. However, algorithms proposed there assumed only usage of numeric procedures and required a lot of preliminary calculations.

In the proposed work the synthesis results of the works [3,4,5] are generalized for the case when the speed on the trajectory is a function of generalized coordinates and speeds.

From our point of view, such an approach allows to formalize the procedure of the time optimal robot's control systems.

2 Task Statement

Assume that mathematical model of manipulating robot's dynamics is presented in the form [2]:

$$A(q)\dot{q} + q^THq + C(q) = \tau$$  (1)

where $A(q)$ - $(n\times n)$ inertia matrix; $H$ -nxnxn coefficients tensor of centrifugal and coriolis components; $C$- $(n\times 1)$ gravitational components; $\tau$ -$(n\times 1)$ control actions vector; $q, q, q$ - $(n\times 1)$ generalized vectors of accelerations, speeds and coordinates respectively; here n – number of MR's degrees of freedom.

Frequently it is expedient to consider the model (1) in the Kochi form. The direct kinematics task solution is added to the Kochi form[3]:

$$\begin{align*}
\dot{x}_{2-i} &= x_{1i} \\
\dot{x}_i &= F(x_{2-i}, x_i) + B(x_{2-i})\tau, \\
P &= \Phi(x_{2-i}), \ i = 1, n
\end{align*}$$  (2)

$$P = \Phi(x_{2-i}), i = 1, n$$  (3)

where $x_{2-i}, x_i$ - $(n\times 1)$ vectors of state coordinates corresponding to the generalized coordinates and speeds; $F = -A^{-1}[H + C]$ - $(n\times 1)$ nonlinear elements vector; $B$ - $(n\times n)$ coefficients matrix; $P$ - $(m\times 1)$ external coordinates vector (its components are smooth solutions of the direct
of the generalized coordinates in the following forms:

\[ T_{i2}x , \quad (4) \]
\[ T_{P} \quad (5), \]

where \( \sum \) - two times differentiable functions of their arguments.

According to the known works the task of time optimal control requires minimization of the integral criterion of the following form:

\[ J = \int_{0}^{1} dt , \quad (6) \]

where \( t_0, t_1 \) - the starting and finishing moments of motion respectively.

In organization of speed optimal process it is required to ensure \( J = J_{opt} = \min \) and to satisfy the following limitations

\[ \tau_{\min} \leq \tau \leq \tau_{\max} . \quad (7) \]

The technological task set above can be formulated in the following way:

For a manipulating robot presented by the mathematical model (1) or (2) it is necessary to synthesized such a control \( \tau(x_{2i-1}, x_{2i}) \) that in motion from an arbitrary initial condition \( x_{2i-1} = x_{2i}(t_0) \) to the set final one \( x_{2i-1} = x_{2i}(\tau_i) \) along the set trajectories (4) or (5) will ensure asymptotic stability of these trajectories and would minimize the criterion (6), i.e. \( J \rightarrow J_{opt} \). The limitation on the control action (7) should be accounted.

3 Trajectories Planning and Control Synthesis

Let's take a closer look at the form of trajectories (4) and (5). It is known that most of the existing methods of planning the motion of the MR's effector operate in the space of external coordinates \( R^m \). The range of these trajectories is frequently limited by the arcs are line segments

\[ \sum_i^{m} = \sum (\rho) , \quad (4) \]
\[ \sum_i^{m} = \sum (x_{2i-1}) , \quad (5) \]

where \( \sum \) = set coefficients matrices, \( \dim M_{ij} = (n \times n) \), \( \dim M_{ij} = (1 \times n) \), \( \dim M_{ij} = (1 \times 1) \), \( 0_1 \) - zero matrix of dimension \((1 \times 1)\).

In particular, if \( m = n = 3 \) two top rows of the vector \( \sum^m_i \) can be presented by the cone equation and the equation of the secant surface setting the trajectories in the form of conical cross-sections [9]. In case of \( m = n = 2 \) the dimension of the vector \( \sum^m_i \) is equal to 2 and its significant element can be presented by an equation of the one of the second order curves (circle, ellipse, hyperbola, parabola or straight line).

Any of the mentioned curves can be converted to canonical form according to the well known procedures [9].

If \( \sum_i^m \) vector \( \sum^m_i \) of \( \rho \) is supplemented by the conditions of the desired effector orientation, by the requirements of the technological task or by the solutions of optimization tasks. It should be mentioned that setting the trajectories as quadratic forms is not a necessary requirement. It is possible to set the trajectories in other forms as a result of using optimization or other procedures.

Now let's formulate requirements to the external speeds. Since the asymptotic stability quality is desired, an obvious requirement is \( \sum^m_i = 0 \), i.e. absence of speed component normal to he trajectory \( \sum^m_i \) (8). It is also obvious that due to the task set it is necessary to reach some speed along the desired trajectory \( \sum^m_i \) (8). In other words the requirements to the trajectory speed can be set in a form of the following equation

\[ \sum_i^{m} = \left| \frac{\sum^n}{p^T \rho - V(p)} \right| = 0 . \quad (9) \]

Now we will not discuss the form of the function \( V(p) \). It is the speed along desired trajectory. Let's just assume
that it is smooth on all the trajectory segment \([p_0, p_f]\) in 
\(R^n\) or on the corresponding segment \([x^0_{2i-1}, x^0_{2i+1}]\) in \(R^n\). 

Since we assumed the function \(\Sigma\) is differentiable and the solutions \(\Phi\) (3) are smooth, the transformations (9) can be presented in the following form:

\[
\sum^\nu = J^\nu J^\nu x^\nu + V = 0,
\]

\[
V = \begin{bmatrix} 0 \\ -V^\nu(x_{2i-1}) \end{bmatrix},
\]

(10)

where \(J^\nu\) — Jacobean of the vector-function (4), \(\dim J^\nu = (n \times n)\),

\[
J^\nu = \begin{bmatrix} J^\nu(x_{2i+1}) \\ \{J^\nu(x_{2i}) \}^T \end{bmatrix}, \quad J^\nu = \frac{\partial \Sigma}{\partial p}(x_{2i-1})
\]

(11)

\(J^\nu\) — Jacobean vector of the function (3), \(\dim J^\nu = (n \times n)\),

\[
J^\nu = \frac{\partial \Phi}{\partial X^\nu},
\]

(12)

0\(_2\) — zero vector of the dimension \((n-1)\times1\).

Obviously [9], the line \(\Psi\) of intersection of manifolds (8) and (10) accounting the expressions (3) determines the desired motion trajectory in the phase space of generalized coordinates:

\[
\Psi = \Sigma^\nu + R \Sigma^\nu = 0,
\]

(13)

where \(R\) — positively determined matrix of set constants, \(\dim R = (n \times n)\).

In the works [3, 4] the procedure of path following regulators is already discussed for \(V^\nu = \text{const}\). They ensure asymptotic stability of trajectories (13) in all the phase space \(R^{\nu \times n}\) of generalized coordinates. Therefore, they also ensure asymptotic stability of the desired motion trajectories (4) and (5) and of the speed manifolds (9) and (10).

Using the procedures stated in these works, we get the path following control algorithm:

\[
\tau = -K^\nu[J^\nu x^\nu + K^\nu],
\]

(14)

where \(\tau\) — positively determined \((n \times n)\) matrix of set constants.

The proposed control algorithm assumes planning the trajectories in the corresponding subspaces \(R^{\nu \times n}\); subsequent transition to the space \(R^{\nu \times n}\) by means of solution of the direct kinematics task (3); synthesis of the control actions according to the expressions (14) that stabilize the desired phase trajectories is all the reachability areas of the corresponding phase coordinates spaces. It should be mentioned that matrix elements \(T\) and \(R\) determine the motion character of the closed-loop system to the manifolds (8) and (9) and should be selected basing on the condition of maximal usage of power capabilities of MR [3]. On the other hand, if the matrix elements \(T\) and \(R\) approach zero, the near optimal control (14) will approach the optimal control. But in these modes the robot's joints can suffer overloads if the initial conditions don't belong to the manifolds (13). So basing on the "softer" requirements to dynamics it is recommended to set the matrices \(T\) and \(R\) with nonzero elements. Naturally, the greater this difference, the farther from optimal becomes the algorithm (14).

### 4 Path Vehicle Forming and Control System Structure

Now let's consider the question of forming the contour speed \(V^\nu\) (10) satisfying the set task.

It is known [10] that the task of speed optimal control motion time minimization is equivalent to motion speed maximization. The procedure of getting the maximum speed profile on the motion trajectory is rather fully stated in the works [6, 7].

Using distance passed along the trajectory \(S(t)\) as its parameter we can present the working coordinates \(q(t)\) as functions depending on \(S(t)\):

\[
q(t) = M(S(t)), \quad q(t_f) = M(S_f),
\]

(15)

\[
q(t_i) = M(S_i),
\]

(16)

where \(t_0, t_f\) — initial and final time moments; \(S_0, S_f\) — initial and final values of function \(S(t)\).

Taking first and second derivatives from the both parts of the firs of Eq. (15) and substituting them into Eq. (1) we get [6]:

\[
D(S)\dddot{S} + L(S)\ddot{S}^2 + C(S) = \tau,
\]

(16)

\[
D(S) = A(S) \frac{dM}{dS},
\]

\[
L(S) = A(S) \frac{dM}{dt} + \left( \frac{dM}{dS} \right)^T H \frac{dM}{dS},
\]

where \(T\) — positively determined \((n \times n)\) matrix of set constants.
where $A, H, C$ — matrices of appropriate dimensions determined in Eq. (1).

Let’s write the equation (16) in the scalar form:

$$d_i(S)\dot{S} + \ell_i(S)\ddot{S}^2 + c_i(S) = \tau_i, i = 1, n$$  \hspace{1cm} (17)

Substituting the inequalities (7) into Eq. (17) we get the following expressions:

$$\tau_{\text{max}} - c_i(S) \leq d_i(S)\dot{S} + \ell_i(S)\ddot{S}^2 \leq \tau_{\text{max}} - c_i(S), i = 1, n$$  \hspace{1cm} (18)

For any junction the $i$th inequality of (18) determines the limited open area formed by a pair of parallel lines on the surface $S^2 S$ for the set value of $S$. The number $n$ of these inequalities form a polygon $\Omega$ [6,7] on the surface.

The robot’s motion can take place only inside the polygon $\Omega$. These polygons have different form for each of the value of $S$.

Obviously the values of allowed trajectory speed $S$ lies between 0 and a certain maximal speed $S_m$. Multitude of all $S_m$ along the trajectory $S(\tau)$ forms the maximal speed curve (MSC) on the surface $S^2 S$. But MSC is determined as maximal abscissa of verges intersection points of polygon $\Omega$ [6,7]. It corresponds to saturation of two actuators and contradicts Chen’s Theorem [11]. Therefore, it is necessary to use $V_i^* = z \cdot S_m^i (x_{2i-1}, x_{2i})$, where $z$ is positive, constant coefficient, $z \to 1$ but $z \neq 1$.

So forming the maximal speed curve $S_m^i(S) = S_m^i (x_{2i-1}, x_{2i})$ and using it in the speed manifold $\Sigma_{\varphi}$ (9) we can ensure control guarantying MR’s maximal speed. Determining the global minimum of the maximal speed function on the interval $[S_0, S_f]$ allows to determine maximal possible constant value of contour speed necessary for organization of laser tailoring, arc welding etc. [3].

Let’s consider the structure presented in Fig. 1 realizing the required algorithm.

According to the inequality (18) the ordinates $S^2$ of the polygon’s vertices can be calculated according to the following expressions:

$$S_m^i(S) = \frac{d_i(\tau_i - g_i) - d_i(\tau_j - g_j)}{d_i(\ell_i) - d_i(\ell_j)}, \hspace{1cm} (19)$$

where $\eta = 1, \mu$, $i = 1, 2n$, $j = 1, 2n$, $i \neq j$

- number of polygon vertices $\Omega$, $n$ — links number.

The expressions (19) are continuously calculated by the computing block (CB) (Fig. 1). Its outputs corresponding to $\eta$-th vertex are fed to the inputs of the matching schemes $MS_1, MS_2, \ldots, MS_\mu$. Simultaneously the Determining the Maximal Vertex Block (DMVB) determines the vertex with a maximal ordinate belonging to the polygon $\Omega$ and gives a constant signal on one of the matching schemes $MS$ corresponding to the maximal vertex.

On the output of this matching scheme and therefore on the input of the Assembly Scheme AS a maximal speed curve is formed. It is formed together with its derivative by Differentiation Block (DB) and then is passed to the control calculation block (CCB). CCB works according to the algorithm (14). Stopping Block (SB) blocks the work of DMVB.
and forms a setting signal \( V_K = 0 \) on CCB if any of the following inequalities is satisfied:

\[
x_{2i-1}(\Delta \xi_i) \leq \frac{K_i}{\tau_{\text{min}}}, \quad i = 1, n,
\]

(20)

where \( \Delta \xi_i \) - stopping parameter of \( i^{th} \) link, \( \Delta \xi_i = S_f - S \), \( K_i \) - kinetic energy of \( i^{th} \) link; \( \tau_{\text{min}} \) - modulus of the control action minimal value. Note that organization of stopping mode is also possible in case of switching on the structure of positioning regulator that is a particular case of the control (14) [3].

The form of the trajectory \( \Sigma_T \) and the corresponding coefficients in the calculating block and SB are determined by the planner P. It should be mentioned that all the calculations are performed as a function of measured phase coordinates \( x_{2i-1} \) (generalized coordinates) and \( x_{2i} \) (generalized speeds).

5 Conclusion

The proposed algorithm of near time optimal control and its structural realization allow to organize motion of MR's effector along the set trajectories with a near maximal contour (trajectory) speed. This doesn't require solution of the inverse kinematics, procedures and there is no need for approximating devices and interpolators. Asymptotic stability at all of the set motion trajectories is guaranteed.

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References


