NARX Models Application to Model Based Nonlinear Control

Andrzej Dzieliński
Institute of Control & Industrial Electronics
Warsaw University of Technology
ul. Koszykowa 75
00-662 Warsaw, Poland
phone: (+48 22) 622 3155, fax: (+48 22) 625 6633
adziel@isep.pw.edu.pl

Keywords: NARX models, nonlinear control, model based control.

Abstract

The paper discusses the applicability of approximate NARX models of nonlinear dynamic systems to model based nonlinear control. The models might be obtained by a new version of Fourier analysis based neural network. The proposed controller is based on a discrete-time model of the plant. The objective is to incorporate plant modelling and control design into a unified framework where on the one hand a priori knowledge on the plant is exploited, and on the other hand the model is explicitly designed to satisfy the requirements of the control design. The output feedback control structures are proposed and the use of input-output plant models, i.e. models of the NARX type is suggested. The control design method employed are robust with respect to plant uncertainties, modelling errors and external disturbances.

1 Introduction

The problem of controlling a nonlinear single-input, single output nonlinear system is widely considered in the literature (see e.g. [7, 13, 10]). In this paper a controller based on a discrete-time neural network model of the plant that has been obtained by identification from sampled input-output data is proposed. The objective is to incorporate plant modelling and control design aspects within a unified framework (see e.g. [7, 13, 10]). In this paper a controller based on a discrete-time model of the plant. The output feedback control structures are proposed and the use of input-output plant models, i.e. models of the NARX type is suggested. The control design method employed are robust with respect to plant uncertainties, modelling errors and external disturbances.

2 NARX Plant Models

A standard approach for representation of a nonlinear plant in discrete time is to use a Nonlinear Autoregressive model with eXogenous inputs (NARX) which relates the output \( y \) at the discrete time instant \( t \) to past outputs and inputs \( u \)

\[
y(t) = f(y(t-1),\ldots,y(t-n),u(t-d),\ldots,u(t-n)) \quad (1)
\]

where \( n \) is the order of the system to be modelled and \( d \) denotes its relative degree (i.e. the number of time steps delay between the input and the output). The advantage of the NARX structure in comparison to the state space model is that it is based on the knowledge of input and output signals only and no information on internal states is required. A disadvantage is that the NARX model is a non-minimal representation of the system dynamics where \( f \) is a function in \( 2n - d + 1 \) variables. This will result in a high dimensional input space. The order of the NARX model is \( 2n - d \). This order may have to be increased if global validity of the model is required.

Neural network techniques can be used to represent \( f \). Typical multi-layer and single-layered structure have been proposed in [8, 11] and also some specific structures like Fourier analysis based neural network in [5]. The resulting network structure is often referred to as feedforward neural network. Although single-layer structures might not achieve the same degree of accuracy in approximating \( f \) they have the advantage that a linear parameterization can be used

\[
f(y(t-1),\ldots,y(t-n),u(t-d),\ldots,u(t-n)) = \phi(\psi(t-1)) \\
\approx \sum_{i=1}^{N} c_i \rho_i(\psi(t-1)) \quad (2)
\]

where

\[
\psi(t-1) = (y(t-1),\ldots,y(t-n),u(t-d),\ldots,u(t-n))
\]

denotes the information vector, the \( \rho_i \) are the basis functions and the parameters \( c_i \) can then be identified from experimental data. An FEM neural network structure of this type has been proposed by Kalokkuhl and Hunt [8] and a frequency
domain approach for feedforward neural network modelling is presented by Dzieliński and Żbikowski [5]. Besides being accessible to standard identification techniques, linearly parameterized networks are the only structures for which adaptive controllers with guaranteed stability properties can be designed.

3 Output Feedback Controllers

3.1 Two-degrees of freedom controller

The usual control applications considered are mainly related to the servomechanism problem, i.e. the plant output is to track a given reference signal. For this control task two-degrees of freedom control structures are required [6, 1] since they allow the two objectives of attaining a desired system response and rejecting the effects of disturbance and plant uncertainties to be addressed simultaneously. A typical two-degrees of freedom controller for nonlinear system is shown in figure 1. It consists of two components:

1. a trajectory generator calculating a feasible (nominal) input-state trajectory \((x_{\text{nom}}, u_{\text{nom}})\) from a command input. This is essentially an open-loop tracking problem. Its solution requires the stable inversion of the nonlinear plant dynamics;

2. a feedback controller to keep the state \(x\) of the plant close to the nominal trajectory \(x_{\text{nom}}\) in the presence of disturbances and uncertainty. This part of the controller should also be nonlinear. In the simplest case the controller parameters can be scheduled according to the nominal trajectory to accommodate changes of the local dynamics of the plant along the trajectory.

The design of both parts of the controller requires nonlinear control design techniques.

3.2 Geometric Internal Model Controller

Much attention in the field of nonlinear control is currently being dedicated to the differential geometric approach. The differential geometric approach provides two basic tools that are potentially useful for the design of nonlinear two-degrees of freedom controllers:

1. model matching, where the output \(y\) of the nonlinear system is asymptotically matched to the output \(y_t\) of a linear reference system.

2. disturbance decoupling with measurement, where a disturbance acting on the nonlinear system is made unobservable with respect to the system output.

These concepts are explained in detail in the textbooks [7, 13]. The relevance for the discrete time internal model control (IMC) problem has been illustrated in [8].

Both model matching and disturbance decoupling techniques can be shown to be strongly related to the problem of calculating the left inverse of the respective system [10]. Geometric controllers are usually state feedback structures. But, if the state of the plant cannot be measured, output feedback has to be applied. Most of the output feedback structures based on geometric methods use state observers. An alternative approach is based on an internal model control which uses a dynamic output error estimation mechanism instead of estimating the states of the plant. This type of strategy was first proposed by Alvarez et al. in its continuous-time version [3] and in its discrete-time version [2] where geometric methods are applied to derive a two-degrees of freedom nonlinear control structure. An analysis of the resulting control system structure and an extension to the control of an unstable systems has been carried out by Liceaga and Kalkkuhl [9, 12].

The structure of the nonlinear IMC controller is shown in figure 2. A unity gain tracking filter \(G_t\) is used to generate a reference output from the command input signal \(z\). An internal (neural) model serves the purpose of predicting the dynamical behaviour of the plant. Let us consider the case of a perfect internal model where no external disturbances act on the plant and where there is no initial state mismatch between plant and model. In this case it would suffice to use a
state feedback controller to track the internal model along the reference output (model matching) and use the control signal $u_m$ generated in this way to control the plant in a feedforward manner $u = u_m$. But, since there will always be uncertainty, feedback is also needed. The uncertainty will manifest itself in the mismatch between plant and model output (modelling error). Consequently, feedback can be obtained by predicting the dynamical behaviour of the modelling error and correcting the control accordingly. Thus, the internal model output is compared with plant output to obtain the modelling error $\epsilon$. To estimate the dynamics of the modelling error, an error estimation filter $G_r$ is used.

Based on the IMC structure, different strategies can be applied in the controller to generate the actual control law. The version proposed by Alvarez et al. [3, 2] applies the same control input to both process and model $u = u_m$ and uses the disturbance decoupling with measurement to decouple an auxiliary output from incoming signals. It has been shown that this strategy results in the two-degrees of freedom structure, where the dynamics inverse of the internal model acts as a precompensator to the plant while uncertainties are counteracted by a Youla parameterized linear controller. Note that this controller will contain an integrator so that a zero steady state error is obtained.

If the internal model is unstable the strategy mentioned above will not lead to an internally stable controller. In such a case the internal model needs to be stabilized. This can be achieved in two different ways:

1. The model can still be run as a predictor but with an additional output-injection loop stabilizing its states. In this way the internal model will be replaced by a nonlinear observer resulting in a conventional state-feedback linearization scheme with an observer. The original IMC philosophy, however, will be lost.

2. Another option is to use independent control inputs for process and internal model and to run the internal model as a generator of nominal trajectories. This can be realized by model matching techniques. The two-degrees of freedom structure is obtained this way.

For the case of an internal model that is linear in the input $u_m(t)$ the latter controller can be decomposed. In this case the principle of the controller becomes much more obvious and the similarity to figure 1 appears. The neural model is used to generate a nominal input-state trajectory $(x_m, u_m)$ from a reference output $y_r$ and the nominal input is then fed forward into the plant. Counteraction of uncertainties and disturbance rejection is done via a linear controller $C$ whose gain is scheduled using the nominal state $x_{nm}$.

Since they are based on the inversion of the internal model, any of the control systems presented so far will be internally stable only if there exists a stable causal inverse of the model. In terms of geometric control this means that the zero dynamics of the model are required to be bounded-input-bounded-output stable. Such a model is called minimum phase.

In the following sections stability issues of the proposed control systems and modifications of the approach in the case of non-minimum phase plants will be discussed.

### 4 Stability problems of the controller

The proposed controller structures should be stable, in particular with respect to perturbations of the initial states, and robust against modelling errors. For the internal model control structure the general stability results for nonlinear IMC apply and necessary conditions are:

1. Existence of unique equilibrium points of the closed loop system such that for a given constant $z$ we obtain the plant output $y_{eq} = z$ at the equilibrium.

2. For any constant input $u^e_m$ there exists a globally asymptotically stable equilibrium point $(y^e_{meq}, u^e_{meq})$ of the internal model. This excludes unstable models.

3. The zero-dynamics of the internal model are BIBS stable (minimum phase).

For the two-degrees of freedom structure an obvious necessary condition is the existence of a fixed stabilizing linear output feedback controller for any given setpoint. Furthermore it is necessary to show that the linear gain scheduled controller stabilizes the plant about the nominal trajectory.

One of the key requirements for the proposed controller structures to be stable is that the internal model be minimum phase. For the discrete-time version of the controller this requirement is particularly restrictive as the minimum phase property might be lost when sampling the continuous-time systems of relative degree greater than two. Hence, since the controller is based on a discrete-time NARX model of the plant, non-minimum phase behaviour might often occur. Our objective with respect to the two-degrees of freedom internal model controller is to solve the inversion problem for the internal NARX model in a stable and causal way and thus obtain an approximate plant inverse. After having obtained the inverse, the second issue is that control systems with non-minimum phase plants require limited bandwidth. Thus, both the regulating and tracking filter have to be tuned appropriately.

A straightforward way to obtain a minimum phase NARX model is to increase the sampling time or to restrict the structure of the model to those that do not have zero-dynamics. This approach might however increase the modelling error and lead to an unsatisfactory model.

An interesting solution for a class of maximum phase nonlinear systems has been presented by Doyle et al. [4]. Maximum phase systems do not have stable or centre manifolds in their unforced zero-dynamics. The maximum phase property is indicated by the fact that the linearization about any equilibrium point $(y^e_{eq}, u^e_{eq})$ has its zeros only outside the unit circle. In this case the reversal of time for the zero-dynamics will lead to a minimum phase system with the zeros of the
linearization being reflected inside the unit circle. For NARX model this means that if
\[ y(t) = f(y(t-1), \ldots, y(t-n), u(t-d), \ldots, u(t-n)) \]
is maximum phase, then
\[ y(t) = f(y(t-1), \ldots, y(t-n), u(t-n), \ldots, u(t-d)) \]
will be minimum phase. The static gain and the relative degree of the system remain unchanged.

5 Conclusions

A new version of the control structure of a SISO nonlinear system is presented. The controller based on a model of a plant given as NARX model obtained from the input-output data was described. The modelling method may be based on multilayer or RBF neural networks. Also the method based on a harmonically-limited N-D Fourier transform, which enables reconstruction of the right-hand side of NARX equation in the multi-dimensional frequency domain via feedforward neural networks can be applied.

The neural models usually suffer from certain degree of inaccuracy. Hence the robustness of the control structure against modelling errors is an important issue. The stability problem of the presented structures is also discussed and the results are presented.

References


