Numerical solution of large scale structured eigenvalue problems

Volker Mehrmann, David Watkins
Fakultät für Mathematik
TU Chemnitz
D-09107 Chemnitz
{mehrmann,watkins}@mathematik.tu-chemnitz.de

Keywords: Skew-Hamiltonian/Hamiltonian pencil, quadratic eigenvalue problem, implicitly restarted Arnoldi method, linear quadratic control.

In this paper we study large sparse generalized eigenvalue problems with skew Hamiltonian/Hamiltonian pencils. Such problems arise in the computation of optimal control problems for partial differential-equations, in finite element analysis for problems in structural mechanics and also in the numerical simulation of the elastic deformation of anisotropic materials.

In most applications one is interested in the eigenvalues closest to the imaginary axis, either in the left or right half plane. Since the large scale problems often come from a finite element discretization typically these eigenvalues have good accuracy, while the ones that are far away from the axis are inaccurate.

We discuss the computation of these eigenvalues and associated eigenvectors using structured Krylov subspace methods which reflect as much as possible the structure of the physical problem, which in this case is a symmetry of the spectrum with respect to both axes of the complex plane and also a pairing of the eigenvectors.

To do this we use a shift-and-invert approach with two different rational functions, a four-shift version of the generalized Cayley transformation that leads to a symplectic operator or a four shift inverse that leads to a skew Hamiltonian matrix. Combining these iterations with structure preserving implicit restart techniques and locking we obtain a very efficient method. We also discuss convergence proofs and implementation details.

We present some numerical examples that prove the superiority of the new structured approach over classical unstructured methods.