Flatness Based Control of a Throttle Plate

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Abstract

In classical gasoline injection systems, the control of the throttle plate position is important for efficiency and emission of the engine. Depending on the current load and speed of the engine the angular position of the throttle plate has to follow a desired trajectory. The tracking problem is treated by using the flatness property of the plate together with different control strategies. Experimental results from a hardware-in-the-loop test stand show that flatness-based control serves well for solving the given control problem.

1 Introduction

The considered system is the throttle plate shown in Fig. 1. The throttle plate consists of two parts: an electrical part (electric motor) and a mechanical part (throttle flap, mechanical parts of the motor, ...). It is assumed that the throttle flap is directly driven by the motor. Therefore, no gearbox model has to be included. Furthermore, the joint between the motor and the throttle flap is rigid.

The system is described by using a model with the state vector \( x = [I, \phi, \dot{\phi}]^T \) and the input \( u \): \( I \) is the motor current in Ampere, \( \phi \) the position of the motor and throttle flap measured in radians and \( \dot{\phi} \) is the angular velocity of the motor and throttle flap measured in radians/second. The input \( u \) is a pulse-width-modulated (PWM) voltage. In the following, the effects of the pulse-width-modulated input can be neglected, because it is assumed that the electrical subsystem is very slow compared to the PWM frequency. The states are given by the following differential equations:

\[
\begin{align}
\dot{x}_1 &= -\frac{R}{L} x_1 - \frac{C_m}{L} x_3 + \frac{1}{L} u \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= \frac{C_m}{J} x_1 - \frac{C_f(x_2)}{J} x_2 - \frac{D}{J} x_3 + M(x_2, x_3).
\end{align}
\]

The coefficients have the following meaning: \( R \) is the resistance of the motor, \( C_m \) is the motor constant, \( L \) describes the inductance of the motor. The constant \( J \) is the moment of inertia, \( C_f(x_2) \) represents the mechanical spring constant, and \( D \) describes the mechanical damping. The torque in (1c) is described as

\[
M(x_2, x_3) = \frac{1}{J} \left( C_f(x_2) \phi_{LHP} - M_{pre}(x_2)ight. \\
\left. - M_{FF}(x_2) \text{sign}(x_3) \right).
\]
Because of the special safety demands in the automotive sector, the electrical throttle plate has a defined position to which the flap returns when the power supply of the electric motor fails or an error of a higher level control occurs. This position $\phi_{LHP}$ is called *limp home position*. A consequence of the special construction of the throttle plate is that some parameters of the state equation (1) of the model are different for angular blade positions over and under the limp home position $\phi_{LHP}$. Thus, one gets the following relations for the spring constant $C_f(x_2)$, the pretension of the springs $M_{pre}(x_2)$ at $\phi_{LHP}$ and for the Coulomb friction $M_{fr}(x_2)$:

\[
C_f(x_2) = \begin{cases} 
C_{f1} : & x_2 > \phi_{LHP} \\
C_{f2} : & x_2 \leq \phi_{LHP} 
\end{cases} 
\]

\[
M_{pre}(x_2) = \begin{cases} 
M_{pre1} : & x_2 > \phi_{LHP} \\
M_{pre2} : & x_2 \leq \phi_{LHP} 
\end{cases} 
\]

\[
M_{fr}(x_2) = \begin{cases} 
M_{fr1} : & x_2 > \phi_{LHP} \\
M_{fr2} : & x_2 \leq \phi_{LHP} 
\end{cases} 
\]

Strictly speaking, the model (1), (2) is non linear. Due to the particular structure of (2), it can be considered as being piecewisely linear for different operation areas.

The control problem for (1), (2) is to follow a desired trajectory $\phi_d(t)$ for the angular position $x_2(t)$. This tracking problem has an elegant solution if the system is differentially flat in the sense of [2, 3, 4, 7, 1].

Roughly speaking, this means that there exists a (fictitious) output $y = \dim y = \dim u$, called a *flat (or linearizing) output*, with the following properties:

(i) The components of $y$ can be calculated from the state variables $x$, the inputs $u$, and a finite number of their time derivatives

\[
y = \Phi \left( x, u_1, \ldots, u_m, \ldots, u_m \right) 
\]

(ii) Any system variable $z$, i.e., the state variables $x$, the input variables $u$, and their time derivatives and functions of them, can be calculated from $y$ and a finite number of its time derivatives

\[
x = \Psi \left( y_1, \ldots, y_1, \ldots, y_m, \ldots, (\beta_m) \right) 
\]

\[
u = \Psi \left( y_1, \ldots, y_1, \ldots, y_m, \ldots, (\beta_m+1) \right)
\]

Since (1), (2) is considered to be a linear system, the flatness property is equivalent to the controllability of the system [2, 4]. The controllability of (1) can be shown by using well-established criteria [6, 5]. Moreover, the flatness based approach that is presented here offers a straight-forward solution to the given tracking problem exploiting the controllability property.

In section 2, the flatness of the system is shown. The flatness property forms the basis for all subsequent open and closed loop schemes. In section 3, suitable trajectories are designed and an open loop control is discussed satisfying input constraints. In section 4, two control schemes for the closed loop are presented: a linear output feedback including the open loop control strategy, and a flatness-based control scheme based on state feedback. Experimental results for the open and closed loop are presented in section 5.

## 2 Flat output

For the model (1), (2), it can be shown that the angle

\[
y = x_2
\]

is a flat output of the throttle plate. In order to show the flatness of the system, all state variables as well as the input have to be expressed as a function of $y$ and a finite number of its time derivatives. By differentiating (5), one gets

\[
x_3 = \dot{y}.
\]

Using the second time derivative of $y$

\[
\ddot{y} = \frac{C_m}{J} x_1 - \frac{C_f(y)}{J} y - \frac{D}{J} \dot{y} + M(y, \dot{y}),
\]

the current $x_1$ can be calculated:

\[
x_1 = \frac{1}{C_m} \left[ C_f(y) y + D \ddot{y} + J \left( \dot{y} - M(y, \dot{y}) \right) \right].
\]

Finally, with the third time derivative

\[
(3) y = \varphi(y, \dot{y}, \ddot{y}, u)
\]

one gets the input

\[
u = \frac{-DL(JM(y, \dot{y}) + C_m x_1 - C_f(y) y) + D^2 \dot{y} - JCR x_1 + C_m^2 \ddot{y} + C_f \dot{y} + \dddot{y} y}{C_m J}.
\]

The last relation describes the *inverse system* from which the open and closed loop control are derived in the following.

## 3 Trajectory Planning and Open loop control

In order to determine the open loop control law, a suitable desired trajectory $y_d(t) = \phi_d(t)$ has to be defined. According to (10), this trajectory must have smooth derivatives up to the order three. To this extent, a polynomial approach or a third-order time-delay element can be used. Both options
have been tested and give similar results. A third-order time-delay element is implemented more easily on a test bench and is used in the following experimental examinations (Section 5). Nevertheless, it might be advantageous to use a polynomial approach, because of the reduced computational effort in the real-time environment. Therefore, the procedure to define a sufficiently smooth desired trajectory

\[ y_d(t) = \sum_{i=0}^{3} k_i t^i, \quad 0 \leq t \leq T \]  

with a suitably chosen transient time \( T \) is described here.

The coefficients \( k_i, i = 0, \ldots, 3 \) of (11) are calculated in such a way that (11) satisfies the conditions

\[
\begin{align*}
y_d(0) &= \phi_0 & y_d(T) &= \phi_T \\
y_d'(0) &= 0 & y_d'(T) &= 0
\end{align*}
\]  

(12)

depending on the transient time \( T \). With (10)–(12), the open loop control can be calculated

\[
u_d = \varphi \left( y_d, y_d', y_d^{(3)} \right). \tag{13}
\]

The structure of the open loop control is shown in Fig. 2.

Since (13) is an explicit expression for the control input, input constraints

\[ u_{\text{min}} \leq u_d(t) \leq u_{\text{max}} \]  

(14)

may be considered off-line. This can be done by a simple evaluation of the functional relation (13) — without requiring the integration of differential equations. If the constraints are violated due to the choice of the transient time \( T \), this parameter has to be increased. Accordingly, it may be decreased to get faster motion in the opposite case. An iterative procedure easily leads to a trajectory and open loop control respecting the constraints with appropriate margins (e.g., 10% of \( u_{\text{max}} \)).

Due to parameter variations in automotive systems and disturbances during operation of a throttle plate, the open loop control by itself is not sufficient to control the throttle plate. Nevertheless, different step responses of the flatness-based open loop control are shown in section 5.1 to document this statement.

4 Closed loop control

To ensure a steady state accurate step response and reduce the influence of parameter variations, the plate has to be operated in closed loop. Using the flatness-based open loop strategy (13), an additional feedback can be determined in order to achieve desired dynamic behaviour and to compensate the external disturbances.

In the following, two possibilities for a closed loop scheme are presented:

1st strategy:

For control of the throttle plate, the flatness-based open loop (13) with an additional \( PI \)-controller can be used. The open loop control \( u_d(t) \) is to impose the desired trajectory to the system, whereas the \( PI \)-controller deals with deviations caused by external disturbances and parameter changes. The design of the \( PI \)-Controller (15) is based on the root-locus method and is primarily optimized to enhance the disturbance reaction of the closed loop. Using an integral part in the structure of the controller

\[
e(t) = \phi_d(t) - \phi(t)
\]

\[
u_{\text{pi}}(t) = K_p e(t) + K_I \int_0^t e(\tau) d \tau, \tag{15}
\]

a steady state accurate step response of the control loop is ensured. The step response of the closed loop control at different steps for the desired value is shown in section 5.1.

Note that the proposed control scheme is a pure output feedback, i.e., no state estimation is necessary.

2nd strategy:

The second control scheme comprises a flatness-based closed loop that can be obtained as follows: For (9), a new input is defined

\[
\tilde{y} = v.
\]  

(16)

By setting

\[
v = y_d + \sum_{k=0}^{2} \frac{p_k}{y_d - y} (k) \tag{17}
\]

the closed loop (1), (10) is stabilized asymptotically by a suitable choice of the coefficients \( p_k, k = 0, 1, 2 \). However, this type of feedback is a state feedback and requires knowledge of the state variables either by measurement or by a state observer.

The reconstruction of the state variables is done on the basis of the measured quantities

\[ \eta_1 = x_1, \quad \eta_2 = x_2. \]  

(18)

In order to reduce the computational effort, no state observer for the estimation of the non-measured state variables \( \hat{\phi} = \phi \)
and $\ddot{y} = \ddot{\phi}$ is used designing the closed loop control. Here-with, the implementation in an electronic control unit is eased. The experimental results in the next section show the validity of this approach.

For this, the first derivative is approximated using the difference

$$\dot{y}_{ap}(t) \approx \frac{\eta_2(t) - \eta_2(t - \Delta t)}{\Delta t}. \quad (19)$$

With this difference, the measurements (18) and (1c), an approximation for the second derivative of $y$ is found

$$\ddot{y}_{ap}(t) \approx \frac{C_m}{J} \eta_1 - \frac{C_f}{J} \eta_2 - \frac{D}{J} \dot{y}_{ap} + M(\eta_2, \dot{y}_{ap}). \quad (20)$$

The scheme of the flatness-based control structure is shown in Fig. 4. In order to achieve steady state accuracy, a $PI$-controller according to (15) is used. Furthermore, the coefficients $p_k$, $k = 0, 1, 2$ of the state feedback controller have to be determined in order to create an acceptable input response of the closed loop. This task can be solved by assigning the eigenvalues of the error dynamics. The eigenvalues of the error dynamics are chosen to be a conjugate complex pair of eigenvalues and an additional one on the real-axis with the following parameters:

$$P_{1,2} = -80 \pm j 30 \left[ \frac{1}{s} \right], \quad P_3 = -100 \left[ \frac{1}{s} \right]. \quad (21a)$$

Based on these eigenvalues, the coefficients $p_k$, $k = 0, 1, 2$ of the state feedback controller are given by the following values:

$$p_0 = 730000 \left[ \frac{1}{s^3} \right], \quad p_1 = 23300 \left[ \frac{1}{s^2} \right], \quad p_2 = 260 \left[ \frac{1}{s^3} \right]. \quad (21b)$$

The experimental results of the closed loop control using a state feedback controller are shown in section 5.3.
5 Experimental results

Traditionally, in the automotive industry step inputs are used to verify the dynamic behaviour of a component like the throttle-plate. That is why different step inputs of the desired value $\phi_d(t)$ have been carried out to present and to compare the responses of the open and closed loop controls. For each control method, the plots of three different step responses of the desired value are shown:

- $\phi(t_1) = 20^\circ$ to $\phi(t_2) = 80^\circ$
- $\phi(t_1) = 45^\circ$ to $\phi(t_2) = 55^\circ$
- $\phi(t_1) = 5^\circ$ to $\phi(t_1) = 65^\circ$

This means that the first two steps are above the limp-home-position, which is located at $12^\circ$. The third step of the desired value shows the input response of the loop when crossing the dominant non-linearity of the system — the limb-home-position. To ensure good comparability of the systems reactions, the step-height is always the same.

For the flatness-based open and closed loop controls the desired trajectory $\phi_d(t)$ is calculated on the basis of $\phi(t)$ described above. This can be interpreted as the movement of the accelerator pedal in conjunction with the trajectory element (third-order time-delay element or polynomial approach – see Section 3).

For a comparison, a step response of a common output feedback with a linear PID-Controller is shown in Fig. 5. These step responses can be considered as state of the art technology and thereby serve as reference for comparison with the step responses of the flatness-based control schemes of the throttle plate. The parameters $K_p$, $K_I$ and $K_D$ of the PID-Controller (22) are designed and optimized using the root-locus method. It can be assumed, that it is not possible to improve the quality of the control significantly when using this control structure

$$u_{pid}(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau + K_d \dot{e}(t).$$  \hspace{1cm} (22)

In Fig. 5, it can be seen that — using the PID-controller — the actual value of the angle $\phi(t)$ approaches the desired value asymptotically. There is no overshooting, although the actual value is oscillating slightly while moving towards the desired value. When increasing the proportional gain $K_p$ of the PID-controller, the oscillation rapidly grows and the control almost reaches the limit of stability. The input response of the closed loop finally achieves steady state accuracy because of the integral part of the controller.

The input response of the PID-controller crossing the LHP is shown in Fig. 6. Note that the linear control across the LHP, the main non-linear part of the throttle plate, obviously leads to a less desirable response of the closed loop. The actual angle of the throttle plate remains at the LHP for 300ms until the integral part of the controller is getting strong enough to move the plate across the LHP (note the stretched time axis). Then, the desired value is approached asymptotically.

![Figure 6: Input response of a closed loop control across the LHP using a PID-Controller.](image)

5.1 Experimental results of the flatness based open loop control

The input response of the open loop control above the limp-home-position (LHP) is shown in Fig. 7. It can be seen that

![Figure 7: Input response of the open loop control above the LHP.](image)
the desired value can not been reached for all times. This is due to model errors and parameter uncertainty. The open loop control is not steady state accurate and the response of the 60° step input shows a permanent angular deviation of 8° (lower limit of $\phi_d(t)$) in respect to 12° (upper limit of $\phi_d(t)$) from the desired value. The targetted dynamics of the desired trajectory, though, is well followed by the actual angle of the throttle plate.

This result is reached using a desired trajectory, which is calculated by a third-order time-delay element as indicated before. It has the time constant $T=40\,\text{ms}$ in combination with a rate limiter of $500^\circ/\text{s}$. This means that a relatively slow desired trajectory is used with the open loop control to reduce saturation effects of the inductance. Thus, the setting voltage $\dot{u}$ does not exceed $6\,\text{V}$ in operation.

The input response of the open loop control is getting even worse if the step-height is getting smaller (right plot of the figure). This is a result of nonlinear friction of the system which is comparatively small and considered a disturbance. Therefore, it is not possible to compensate the stiction of the throttle plate with an open loop control. Furthermore, both figures demonstrate, that the open loop control is very sensitive towards even slight changes of system parameters.

The step of the desired value crossing the LHP and the reaction of the flatness-based open loop control is shown in Fig. 8. It is shown, that the influence of the LHP can be compensated to a great extent using the flatness-based open loop control. Compared to the input response of the linear $PID$-controller (6) the actual value only stays 130ms at the LHP before reaching the desired value below the LHP asymptotically.

5.2 Experimental results of the flatness-based output feedback control

To improve the steady state accuracy and disturbance reaction of the open loop control, a linear feedback controller with integral part is used. It is shown that the input response of this closed loop is improved considerably (Fig. 9) using the same desired trajectory. Thus, the maximum level of the setting voltage is increased slightly compared to the open loop control, but does not reach the input constraints. This graph shows that the actual value of the angle $\phi(t)$ overshoots the desired value. This happens independently of the step height and leads to an asymptotical approach of the actual value $y(t)$ towards the desired value $\phi_d(t)$. Furthermore, the actual value follows the desired value more accurately with this control concept than with the open loop control.

As the parameters of the linear controller (15) are not optimized thoroughly, the reactions to both step-inputs may be improved slightly. A problematic circumstance in this case is the tendency of the integral part of the controller to destabilize the closed loop if not designed carefully. For this reason, the absolute value of the controllers integral part could not be increased significantly. Furthermore, the proportional gain of the linear controller has to stay small in relation to the integral part for stability reasons, as well. These circumstances restrict the performance of an output feedback with a linear $PI$-control.

The step of the desired value crossing the LHP and the response of the flatness-based closed loop control is shown in Fig. 10.

In Fig. 10, it is shown that it is not possible to compensate the discontinuity of the spring characteristics at the LHP and improve the input response with a linear $PI$-controller. Al-
though the time that the throttle plate stayed at the LHP is reduced to 50ms, the use of this linear controller leads to a fairly large under- and overshooting of the actual value. Therefore, it has to be noticed, that the input response of this closed loop crossing the LHP is not improved compared to the flatness-based open loop control (Fig. 8).

5.3 Experimental results of the flatness-based state feedback control

In order to solve the evident problems of the linear position control and the flatness-based output feedback control, a state feedback controller is used in addition to the output feedback [8]. In Fig. 11, the step inputs of the desired value above the LHP are shown. Because of the improved dynamics due to the state-feedback control, the time constant (note the time axes) of the desired trajectory is reduced from 40ms to 12ms. Additionally, the rate limit is increased to 600°/s. Using this desired trajectory, the setting voltage reaches the input limit at 12V.

Figure 11: Input response of the flatness-based closed loop control above the LHP.

According to Fig. 11, it is obvious that the input response of the closed loop using a state feedback controller is enhanced. The actual value of the angle follows the desired trajectory very accurately. The actual value is accurate at steady-state conditions and although the desired trajectory is faster, there is no overshooting. As well, the input response of the closed loop does not depend on the step-height. Since this flatness-based state control is able to follow the desired trajectory, the dynamic qualities of the closed loop are defined by the choice of the transient time. The dynamics are primarily limited by the physical properties of the throttle plate (motor-torque, moment of inertia, etc.). With this desired trajectory, the throttle plate needs 175ms for a step-height of 60° without overshooting. This value can certainly be decreased in further examinations, if necessary.

The input response of the closed loop using a desired trajectory across the LHP is shown in Fig. 12. In this graph it has to be noticed, that the step response is improved compared to the linear feedback control, but it doesn’t follow the trajectory as accurately as the step responses above the LHP. This result, caused by the physical properties, might be improved by different enhancements of the control concepts and is topic of actual research efforts.

6 Conclusions

For the control of a throttle plate, an open and closed loop strategy have been developed on the basis of a nonlinear model. It has been shown, that a flatness-based open loop control on its own is not suitable to control the position of the nonlinear throttle plate. This is demonstrated by different step responses which showed a distinct deviation of steady state accuracy and a strong sensitivity towards variations of system parameters.

For the closed loop control, two different control strategies have been developed. First, a flatness-based output feedback control using a linear controller with integral part (PI-controller) is examined. This closed loop structure shows an improved input response compared to the open loop strategy. Because of the PI-controller, the input response of the closed loop reaches steady state accuracy. This structure shows slight overshooting of the actual value independently of the step-height followed by an asymptotical approach towards the desired value. It was not possible to improve the dynamics of the input response of the system and enhance the plate’s response when crossing the so-called limp-home-position of the throttle flap.

Therefore, a flatness-based state feedback control structure in combination with a linear output feedback has been designed. This strategy shows the best experimental results using the same step inputs of the desired value. Because of the increased dynamic ability by using this control structure, the desired trajectory could be designed a lot faster. In the hardware-in-the-loop environment, this flatness-based control of the throttle plate follows the desired trajectory very accurately and fulfills the stability and dynamic requirements.

References

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