Theorem (a) Any homomorphism $\Phi : \mathbf{Z}_n \to G$, where G is an arbitrary group, is completely determined by $\Phi(1)$. Indeed, if $\Phi(1) = g$, then the homomorphism is given by $\Phi(j) = g^j$.

(b) If $g \in G$, then the map $\Phi(j) = g^j$ defines a homomorphism if and only if $ord(g)|n$, i.e. the order of g must divide n.

Example Find all homomorphisms from $\mathbf{Z}_{18}$ to $\mathbf{Z}_{10}$.

Solution. We calculate the orders of all the elements in $\mathbf{Z}_{10}$. There is one element of order 1 (namely 0) and one element of order 2 (namely 5). All the other elements have order 5 or 10, which do not divide 18. Hence the only possible homomorphisms are $\Phi_1(x) = 0 \bmod 10$ and $\Phi_2(x) = 5x \bmod 10$, i.e. we have two homomorphisms from $\mathbf{Z}_{18}$ into $\mathbf{Z}_{10}$.