

Final topics for MATH 3C

Algebra

You should know how to:

- [§1.2, §1.3, §4.1, §5.1, §5.2, §5.3, §6.1] evaluate or simplify expressions involving numbers/variables and addition, subtraction, multiplication, division, arbitrary exponents, roots, logarithms and the absolute value function. This includes knowing the order of operations and how to add/multiply fractions.
- [§1.2, §1.3, §4.1, §5.1, §5.2, §5.3, §6.1] solve equations in one variable involving addition, subtraction, multiplication, division, exponentiation, logarithms and the absolute value function.
- [§1.3] manipulate inequalities (this includes multiplication by negative numbers and taking reciprocals of both sides).
- [§1.3] find the set of solutions of an inequality in one variable involving addition, subtraction, multiplication, division, exponentiation, logarithms and the absolute value function.
- [§2.3] “complete the square,” i.e. write a quadratic expression $ax^2 + bx + c$ in the form

$$a(x + d)^2 + e.$$

for some constants d and e .

- [§2.3] solve quadratic equations of the form $ax^2 + bx + c = 0$, either by completing the square and simplifying, or by using the quadratic formula. This includes equations with only one solution, and those with no solutions.
- [§4.2] compute the sum/difference/product/composition of two or more polynomials.
- [§4.2] determine the degree of a polynomial, and know how degrees behave when taking sums or products.
- [§4.3] find the zeros of a polynomial of degree higher than two, given information about some of the zeros, or given factors of the polynomial with degree two or lower.
- [§4.3] rewrite a rational function as the sum of a polynomial and a “proper” rational function, i.e. one for which the numerator has lower degree than the denominator. This is the same as dividing two polynomials with remainder.
- [§7.1] solve simple systems of equations using substitution.
- [§7.2] solve systems of linear equations (with either one or no solutions) using gaussian elimination.

Graphs

You should be able to:

- [§1.3] read and write using interval notation (e.g. $(3, 7)$, $[-2, 4]$, $(-\infty, -100]$, etc.), set comprehensions (e.g. $\{x : 3 < x < 7\}$) and unions (e.g. $(-\infty, 0) \cup (0, \infty)$).
- [§2.1] plot a point in the plane given its coordinates, and determine the coordinates of a given point (assuming the axes are labelled and the picture is to scale).
- [§2.1, §2.2, §2.3, §3.2, §3.5, §4.3, §5.1, §5.2, §9.3, §9.4, §10.1, §11.2] sketch graphs of simple equations in two variables, and of functions obtained by transforming elementary functions.
- [§2.1] compute the length of a line segment in the plane, i.e. the distance between two points given their coordinates.
- [§2.1] describe the length of part of a circle in terms of its radius and the number π .
- [§2.2] determine the slope of the line passing through two points in terms of their coordinates.
- [§2.2] determine the slope of a line parallel or perpendicular to a given line.
- [§2.2] find the equation of a line, given its slope and the coordinates of one point on the line. These equations should have the form $y = mx + b$ for some numbers m (the slope) and b (the y -intercept).
- [§2.3] determine the vertex of a parabola, given its equation or a good picture of the graph.
- [§2.3] determine the radius and center of a circle, given its equation or a good picture of the graph.
- [§4.2, §4.3] describe roughly the behaviour of a polynomial or rational function for large values of $|x|$.
- [§4.3] find vertical and horizontal asymptotes for the graph of a rational function.

Functions

You should know how to:

- [§3.1] explain what a function is (or at least, know the difference between a function and an equation or formula).
- [§3.1] determine whether a subset of the plane (e.g. a graph) determines a function, either by definition or using the vertical line test.
- [§3.1] read and write using function notation (e.g. $f(x)$ for a function f) and be able to evaluate f at specific numbers (e.g. $f(2)$) or longer expressions (e.g. $f(\frac{y-5}{2})$).

- [§1.2, §3.1, §4.1, §4.3, §5.1, §5.2, §9.4, §10.1] determine the (largest possible) domain of a function defined by a formula.
- [§1.3, §3.1] determine the range of a function defined by a simple formula, given its domain.
- [§3.1] determine the domain and range of a function given its graph.
- [§3.3] define the sum, difference, product, ratio or composition of two functions, and describe their domains in terms of the original functions.
- [§3.3] read and write using the notation $f \circ g$ for the composition of two functions f and g .
- [§3.3] decompose a complicated function into a composition/sum/difference/product/ratio of two or more simpler functions.
- [§3.2] translate and scale the graph of a function, in both the vertical and horizontal directions, by composing it (on either side) with linear functions.
- [§3.2] given a function f , determine a formula for another function (in terms of f) whose graph is obtained by translating and scaling the graph of f .
- [§3.5] determine intervals on which a given function is increasing or decreasing.
- [§3.4] determine whether a given function is one-to-one, either by definition or using the horizontal line test.
- [§3.4] compute a formula for the inverse of a one-to-one function defined by a simple formula.
- [§3.4] relate the domain and range of a function to that of its inverse.
- [§3.5] determine the graph of a function from the graph of its inverse (or, most likely, vice versa).

Applications

You should know how to:

- [§5.2] give a formula for exponential decay in terms of the initial quantity and its half-life.
- [§5.4, §6.3] give a formula for exponential growth in terms of the initial quantity and some information about the growth rate (including continuous growth rates).
- [§5.4, §6.3] give a formula for compound interest paid in regular intervals at a certain rate, and continuously compounded interest paid at a certain rate.

Trigonometry

You should be able to:

- [§9.1, §9.2, §9.3, §9.4, §10.1] convert between angles in degrees, angles in radians, points on the unit circle, x -coordinates in $[-1, 1]$, y -coordinates in $[-1, 1]$, and slopes (possibly given some additional information to resolve any ambiguities), in terms of trigonometric functions.
- [§9.1, §9.2, §9.6, §10.2] do the above without a calculator for special angles/lengths/slopes, including 0° , 30° , 45° , 60° and their analogues in the other three quadrants.
- [§9.1, §9.2] find the arc length or area of the segment of the circle swept out by a given angle.
- [§9.5, §10.1] determine the side lengths and internal angles of a right triangle, given two lengths or one length and one angle.
- [§3.2, §11.2] describe the amplitude, period and phase shift of a scaled/shifted trigonometric function
- [§3.2, §11.2] construct a periodic function with prescribed amplitude, period, phase shift, etc.