Midterm 1 topics for MATH 3C

Algebra

You should know how to:

- evaluate or simplify expressions involving numbers/variables and addition, subtraction, multiplication, division, integer exponents, square roots and the absolute value function. This includes knowing the order of operations and how to add/multiply fractions.
- solve equations in one variable involving addition, subtraction, multiplication, division and the absolute value function.
- manipulate inequalities (this includes multiplication by negative numbers and taking reciprocals of both sides).
- find the set of solutions of an inequality in one variable involving addition, subtraction, multiplication, division and the absolute value function.
- "complete the square," i.e. write a quadratic expression ax^2+bx+c in the form

  $$a(x+d)^2+e.$$

  for some constants d and e.
- solve quadratic equations of the form $ax^2+bx+c=0$, either by completing the square and simplifying, or by using the quadratic formula. This includes equations with only one solution, and those with no solutions.

Graphs

You should be able to:

- read and write using interval notation (e.g. $(3,7), [-2,4], (-\infty,-100]$, etc.), set comprehensions (e.g. $\{x : 3 < x < 7\}$) and unions (e.g. $(-\infty,0) \cup (0,\infty)$).
- plot a point in the plane given its coordinates, and determine the coordinates of a given point (assuming the axes are labelled and the picture is to scale).
- sketch graphs of simple equations in two variables, and of functions obtained by transforming elementary functions as in §3.2.

- compute the length of a line segment in the plane, i.e. the distance between two points given their coordinates.
- describe the length of part of a circle in terms of its radius and the number π.
- determine the slope of the line passing through two points in terms of their coordinates.
- determine the slope of a line parallel or perpendicular to a given line.
- find the equation of a line, given its slope and the coordinates of one point on the line. These equations should have the form $y = mx + b$ for some numbers m (the slope) and b (y-intercept).
- determine the vertex of a parabola, given its equation or a good picture of the graph.
- determine the radius and center of a circle, given its equation or a good picture of the graph.

Functions

You should know how to:

- explain what a function is (or at least, know the difference between a function and an equation or formula).
- determine whether a subset of the plane (e.g. a graph) determines a function, either by definition or using the vertical line test.
- read and write using function notation (e.g. $f(x)$ for a function f) and be able to evaluate f at specific numbers (e.g. $f(2)$) or longer expressions (e.g. $f(\frac{y-5}{2})$).
- determine the (largest possible) domain of a function defined by a formula.
- determine the range of a function defined by a simple formula, given its domain.
- determine the domain and range of a function given its graph.
- define the sum, difference, product, ratio or composition of two functions, and describe their domains in terms of the original functions.
- read and write using the notation $f \circ g$ for the composition of two functions f and g.
- decompose a complicated function into a composition/sum/difference/product/ratio of two or more simpler functions.

- translate and scale the graph of a function, in both the vertical and horizontal directions, by composing it (on either side) with linear functions.
- given a function f, determine a formula for another function (in terms of f) whose graph is obtained by translating and scaling the graph of f.
- determine intervals on which a given function is increasing or decreasing.
- determine whether a given function is one-to-one, either by definition or using the horizontal line test.
- compute a formula for the inverse of a one-to-one function defined by a simple formula.
- relate the domain and range of a function to that of its inverse.
- determine the graph of a function from the graph of its inverse (or, most likely, vice versa).