

# Midterm 2 topics for MATH 3C

## Algebra

You should know how to:

- evaluate or simplify expressions involving numbers/variables and addition, subtraction, multiplication, division, **arbitrary exponents, roots, logarithms** and the absolute value function. This includes knowing the order of operations and how to add/multiply fractions.
- solve equations in one variable involving addition, subtraction, multiplication, division, **exponentiation, logarithms** and the absolute value function.
- manipulate inequalities (this includes multiplication by negative numbers and taking reciprocals of both sides).
- find the set of solutions of an inequality in one variable involving addition, subtraction, multiplication, division, **exponentiation, logarithms** and the absolute value function.
- “complete the square,” i.e. write a quadratic expression  $ax^2 + bx + c$  in the form

$$a(x + d)^2 + e.$$

for some constants  $d$  and  $e$ .

- solve quadratic equations of the form  $ax^2 + bx + c = 0$ , either by completing the square and simplifying, or by using the quadratic formula. This includes equations with only one solution, and those with no solutions.
- **compute the sum/difference/product/composition of two or more polynomials.**
- **determine the degree of a polynomial, and how it behaves when taking sums or products.**
- **find the zeros of a polynomial of degree higher than two, given information about some of the zeros, or given factors of the polynomial with degree two or lower.**
- **rewrite a rational function as the sum of a polynomial and a “proper” rational function, i.e. one for which the numerator has lower degree than the denominator. This is the same as dividing two polynomials with remainder.**
- **solve simple systems of equations using substitution.**
- **solve arbitrary systems of linear equations using gaussian elimination.**

# Graphs

You should be able to:

- read and write using interval notation (e.g.  $(3, 7)$ ,  $[-2, 4]$ ,  $(-\infty, -100]$ , etc.), set comprehensions (e.g.  $\{x : 3 < x < 7\}$ ) and unions (e.g.  $(-\infty, 0) \cup (0, \infty)$ ).
- plot a point in the plane given its coordinates, and determine the coordinates of a given point (assuming the axes are labelled and the picture is to scale).
- sketch graphs of simple equations in two variables, and of functions obtained by transforming elementary functions as in §3.2.
- compute the length of a line segment in the plane, i.e. the distance between two points given their coordinates.
- describe the length of part of a circle in terms of its radius and the number  $\pi$ .
- determine the slope of the line passing through two points in terms of their coordinates.
- determine the slope of a line parallel or perpendicular to a given line.
- find the equation of a line, given its slope and the coordinates of one point on the line. These equations should have the form  $y = mx + b$  for some numbers  $m$  (the slope) and  $b$  (the  $y$ -intercept).
- determine the vertex of a parabola, given its equation or a good picture of the graph.
- determine the radius and center of a circle, given its equation or a good picture of the graph.
- **describe roughly the behaviour of a polynomial or rational function for large values of  $|x|$ .**
- **find vertical and horizontal asymptotes for the graph of a rational function.**

# Functions

You should know how to:

- explain what a function is (or at least, know the difference between a function and an equation or formula).
- determine whether a subset of the plane (e.g. a graph) determines a function, either by definition or using the vertical line test.

- read and write using function notation (e.g.  $f(x)$  for a function  $f$ ) and be able to evaluate  $f$  at specific numbers (e.g.  $f(2)$ ) or longer expressions (e.g.  $f(\frac{y-5}{2})$ ).
- determine the (largest possible) domain of a function defined by a formula.
- determine the range of a function defined by a simple formula, given its domain.
- determine the domain and range of a function given its graph.
- define the sum, difference, product, ratio or composition of two functions, and describe their domains in terms of the original functions.
- read and write using the notation  $f \circ g$  for the composition of two functions  $f$  and  $g$ .
- decompose a complicated function into a composition/sum/difference/product/ratio of two or more simpler functions.
- translate and scale the graph of a function, in both the vertical and horizontal directions, by composing it (on either side) with linear functions.
- given a function  $f$ , determine a formula for another function (in terms of  $f$ ) whose graph is obtained by translating and scaling the graph of  $f$ .
- determine intervals on which a given function is increasing or decreasing.
- determine whether a given function is one-to-one, either by definition or using the horizontal line test.
- compute a formula for the inverse of a one-to-one function defined by a simple formula.
- relate the domain and range of a function to that of its inverse.
- determine the graph of a function from the graph of its inverse (or, most likely, vice versa).
- **find the domain and range of a function involving exponentiation and logarithms.**

## Applications

You should know how to:

- **give a formula for exponential decay in terms of the initial quantity and its half-life.**
- **give a formula for exponential growth in terms of the initial quantity and some information about the growth rate (including continuous growth rates).**

- **give a formula for compound interest paid in regular intervals at a certain rate, and continuously compounded interest paid at a certain rate.**