Math 10A **Final Examination v. A** **Name:** ____________

December 3, 2016 **(Total Points: *50*)** **PID:** ____________

Instructions

1. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
2. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
3. **Read each question carefully and answer each question completely.**
4. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
5. **Write Name at the top of each page.**

(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Let $f(x) = \sqrt[3]{x}$.

(a) Find the tangent line approximation for f near $x = 27$, using the fact that $3^3 = 27$.

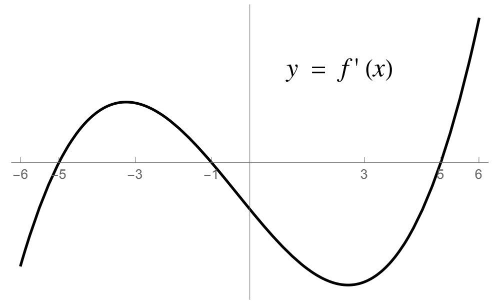
(b) Find a linear approximation for $\sqrt[3]{26}$.
Express your answer as a rational number (a quotient); do not try to "simplify" it.

(6 points) 2. The function $f(x) = Cx^3 + 2x^2 - 30$ has an extreme point (that is, a local maximum or local minimum) at $x = -\frac{1}{3}$,

(a) Find C.

(b) Is the extreme point at $x = -\frac{1}{3}$ a local maximum or local minimum? Justify your answer.

(c) Find the x-coordinate of any inflection point(s) the function may have.



(6 points) 3. The graph $y = f'(x)$, the **derivative** of a function f defined on the interval $-6 \leq x \leq 6$, is shown above. Answer each question using *integers* (that is, *whole numbers*).
No explanation is required.

(a) On which interval(s) is f increasing?

(b) On which interval(s) is f concave up?

(c) List the x-coordinate(s) of the local minima of f.

(d) List the x-coordinate(s) of the local maxima of f.

(e) List the x-coordinate(s) of the inflection point(s) of f.

 Name: ______________________

(6 points) 4. Let $f(x) = \sqrt{x}$. Using the definition of the derivative, evaluate $f'(2)$.

Note: In order to earn credit, you must algebraically evaluate the limit specified by the definition of the derivative. Applying a differentiation formula will not earn any credit.

(6 points) 5. Consider the curve defined by the equation

$$2^{x\,y} = 1 + x^2.$$

Find an equation for the tangent line to the curve at the point $(1, 1)$.

 Name: ____________________

(6 points) 6. Consider the function $f(x) = x^3(x-1)$.

(a) Find all critical points of f.

(b) For each critical point of f, determine if the critical point is a local maximum, local minimum, or neither.

(c) Find all inflection points of f.

 Name: ____________________

(6 points) 7. An international candy company, *Longo's Bubblegum Emporium* (*LBE*), finds that the quantity, q, of sticks of bubblegum sold is a function of the price, p, per stick of gum. The company's revenue, R, is therefore given by the equation $R = q\,p$. Moreover, *LBE* finds that

$$q = 1000\,e^{-p},$$

where p is measured in dollars.

(a) At what price should the candy company *LBE* sell its bubblegum in order to maximize its revenue?

(b) What is the maximum total revenue *LBE* can earn?

(6 points) 8. Evaluate the following limits.

(a) $\lim_{x \to 0} \frac{\sin(x)}{x}$

(b) $\lim_{x \to 0} \frac{\cos(x) - 1}{x}$

(c) $\lim_{x \to 0} \frac{\ln(1 + x)}{x}$