Math 20C **Final Examination** **Name:** ____________________

Spring 2015 **Version A** **PID:** ____________________

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order as they appear in the exam.**
   (c) **Start each numbered problem on a new side of a page.**
6. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
7. **Write Name & PID on this exam sheet and return inside front cover of your Blue Book.**

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0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Find an equation of the plane containing the two parallel lines

$$\ell_1(t) = \langle 1,1,1\rangle + t\langle 1,2,3\rangle \quad \text{and} \quad \ell_2(t) = \langle 1,2,3\rangle + t\langle 1,2,3\rangle.$$

   Write your answer in the form $Ax + By + Cz = D$.

2. (6 points) Find a vector equation for the line of intersection formed by the intersection of the two planes

$$2x - y + z = 5 \quad \text{and} \quad x + y - z = 1.$$

3. (6 points) The light intensity (measured in *lux*) in a certain pond at point (x, y, z) is given by

$$f(x,y,z) = e^{-x/2} + e^{-y/3} + e^{-z/4}.$$

   A South American electric eel is swimming in the pond along a path with position after t seconds given by the function $\mathbf{c}(t) = \langle x(t), y(t), z(t)\rangle$. Suppose $\mathbf{c}(5) = \langle 1, 0, 2\rangle$ and $\mathbf{c}'(5) = \langle 3, 9\pi, 2\rangle$. How fast is the light intensity increasing along the eel's path after 5 seconds?

4. (6 points) A Higgs boson (an elementary particle) is moving along a helical path given by the equations

$$x(t) = 5\cos(2t), \quad y(t) = t, \quad z(t) = 5\sin(2t).$$

   (a) Find the distance traveled by the particle between $t = 0$ and $t = 2\pi$.
   (b) Find the displacement between the particle's position at $t = 0$ and its position at $t = 2\pi$.

5. (6 points) A triangle has vertices $A = (0,0,0)$, $B = (1,2,3)$, and $C = (-3,-1,3)$. Find the three angles of the triangle. Leave your answers in terms of arccos or $\cos^{-1}$.

(Please turn over.)

(This exam is worth 50 points.)

6. (6 points) The gradient of the function $g(x, y, z)$ at $(-3, 4, 5)$ is the vector $\langle -2, 1, 2\rangle$.

   (a) Find the values of g_x and g_y and g_z at $(-3, 4, 5)$.

   (b) Find the maximum rate of change of $g(x, y, z)$ at $(-3, 4, 5)$.

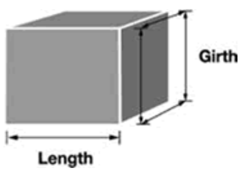
   (c) Find the rate of change of g at $(-3, 4, 5)$ in the direction of the point $(-1, 8, 1)$.

7. (6 points) Evaluate the iterated integral

$$\int_{y=0}^{2} \int_{x=y/2}^{1} y e^{x^3}\, dx\, dy.$$

   Change the order of integration, if necessary.

8. (6 points) Lagrangian Airlines limits the size of carry-on bags by requiring the length of the bag plus the girth (see below) to be less than or equal to 90 inches. Find the dimensions of the bag with the largest volume that can be taken on the plane. (Assume the carry on is a rectangular box, like in the picture below.)



Girth is the distance all the way around the bag moving perpendicular to the length.