Math 142A **Final Examination v. A** **Name:** ____________________

December 15, 2017 **(Total Points: *50*)** **PID:** ____________________

Instructions

1. **Write your Name and PID in the spaces provided above.**
2. **Make sure your Name is on every page.**
3. **No calculators, tablets, phones, or other electronic devices are allowed during this exam.**
4. **Put away ANY devices that can be used for communication or can access the Internet.**
5. **You may use one handwritten page of notes, but no books or other assistance during this exam.**
6. **Read each question carefully and answer each question completely.**
7. **Write your solutions clearly in the spaces provided.**
8. **Show all of your work. No credit will be given for unsupported answers, even if correct.**

(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions given before the exam or written on the chalkboard during the exam.

(6 points) 1. Let $\{a_n\}$ be a monotonically increasing sequence with a subsequence $\{a_{n_k}\}$ such that $a_{n_k} \to a$. Prove that $a_n \to a$.

(6 points) 2. Let $U \subseteq \mathbb{R}$ be a closed subset of $\mathbb{R}$ that is dense in $\mathbb{R}$. Prove that $U = \mathbb{R}$.

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(6 points) 3. Let $f : \mathbb{R} \to \mathbb{R}$ be continuous at x_0 with $f(x_0) > 0$. Prove that there is a natural number n for which $f(x) > 0$ for all x in the interval $I := (x_0 - 1/n,\ x_0 + 1/n)$.

(6 points) 4. Suppose S is a sequentially compact subset of $\mathbb{R}$ and that $f : S \to \mathbb{R}$ is continuous. Prove that f attains its maximum value. [Note: A sequentially compact set need not be an interval; thus, this generalizes the Extreme Value Theorem in your textbook to arbitrary sequentially compact sets.]

(6 points) 5. Let M be a natural number. Prove that $S_M = \left\{ \frac{1}{m} \,\middle|\, m \in \mathbb{N} \text{ and } 1 \le m \le M \right\}$ is sequentially compact.

(6 points) 6. Suppose $f : (a,b) \to \mathbb{R}$ is differentiable and that $f' : (a,b) \to \mathbb{R}$ is bounded. Prove that $f : (a,b) \to \mathbb{R}$ is uniformly continuous.

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(6 points) 7. Prove that any function $f : \mathbb{Z} \to \mathbb{R}$ is uniformly continuous.

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(6 points) 8. Suppose $f : \mathbb{R} \to \mathbb{R}$ has three derivatives and also satisfies the following three conditions:

(1) There is a constant $M > 0$ such that $|f(x)| \leq M\,|x|^3$ for all x,

(2) $f(0) = 0$, and

(3) $f'(0) = 0$.

Show that $f''(0) = 0$.