Instructions

1. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
2. You may use one handwritten page of notes, but no books or other assistance during this exam.
3. Read each question carefully and answer each question completely.
4. Show all of your work. No credit will be given for unsupported answers, even if correct.
5. Write Name at the top of each page.

(2 points) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Let $f(x) = \sqrt{x}$.
   
   (a) Find the tangent line approximation for $f$ near $x = 27$, using the fact that $3^3 = 27$.

   (b) Find a linear approximation for $\sqrt{26}$.
       
       Express your answer as a rational number (a quotient); do not try to “simplify” it.
2. (6 points) The function $f(x) = Cx^3 + 2x^2 - 30$ has an extreme point (that is, a local maximum or local minimum) at $x = -\frac{1}{3}$.

(a) Find $C$.

(b) Is the extreme point at $x = -\frac{1}{3}$ a local maximum or local minimum? Justify your answer.

(c) Find the $x$-coordinate of any inflection point(s) the function may have.
(6 points) 3. The graph \( y = f'(x) \), the derivative of a function \( f \) defined on the interval \(-6 \leq x \leq 6\), is shown above. Answer each question using integers (that is, whole numbers). No explanation is required.

(a) On which interval(s) is \( f \) increasing?

(b) On which interval(s) is \( f \) concave up?

(c) List the \( x \)-coordinate(s) of the local minima of \( f \).

(d) List the \( x \)-coordinate(s) of the local maxima of \( f \).

(e) List the \( x \)-coordinate(s) of the inflection point(s) of \( f \).
(6 points) 4. Let \( f(x) = \sqrt{x} \). Using the definition of the derivative, evaluate \( f'(2) \).

\textbf{Note:} In order to earn credit, you must algebraically evaluate the limit specified by the definition of the derivative. Applying a differentiation formula will not earn any credit.
(6 points) 5. Consider the curve defined by the equation

\[ 2^x y = 1 + x^2. \]

Find an equation for the tangent line to the curve at the point \((1,1)\).
6. Consider the function $f(x) = x^3(x - 1)$.

(a) Find all critical points of $f$.

(b) For each critical point of $f$, determine if the critical point is a local maximum, local minimum, or neither.

(c) Find all inflection points of $f$. 
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(6 points) 7. An international candy company, *Longo’s Bubblegum Emporium (LBE)*, finds that the quantity, $q$, of sticks of bubblegum sold is a function of the price, $p$, per stick of gum. The company’s revenue, $R$, is therefore given by the equation $R = qp$. Moreover, *LBE* finds that

$$q = 1000e^{-p},$$

where $p$ is measured in dollars.

(a) At what price should the candy company *LBE* sell its bubblegum in order to maximize its revenue?

(b) What is the maximum total revenue *LBE* can earn?
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(6 points) 8. Evaluate the following limits.

(a) \( \lim_{x \to 0} \frac{\sin(x)}{x} \)

(b) \( \lim_{x \to 0} \frac{\cos(x) - 1}{x} \)

(c) \( \lim_{x \to 0} \frac{\ln(1 + x)}{x} \)