1. Let $A$, $B$, and $X$ be $n \times n$ matrices such that $A$, $X$, and $A - AX$ are invertible with $(A - AX)^{-1} = X^{-1}B$.
   (a) Explain why $B$ is invertible.
   (b) Solve the equation $(A - AX)^{-1} = X^{-1}B$ for $X$. If you need to invert a matrix, explain why that matrix is invertible.

2. Let $A$ be an invertible matrix. Explain why the columns of $A^{-1}$ are linearly independent.

3. Let $W$ be the set of all vectors of the form
   \[
   \begin{bmatrix}
   s + 3t \\
   s - t \\
   2s - t \\
   s + t
   \end{bmatrix}.
   \]
   (a) Show that $W$ is a subspace of $\mathbb{R}^4$.
   (b) Let $v = \begin{bmatrix} 9 \\ 1 \\ 4 \\ 5 \end{bmatrix}$. Determine whether or not $v \in W$.

4. Let
   \[
   A = \begin{bmatrix}
   1 & 5 & -4 & -3 & 1 \\
   0 & 1 & -2 & 1 & 0 \\
   0 & 0 & 0 & 0 & 0
   \end{bmatrix}.
   \]
   Find a linearly independent set of vectors that span $\text{Nul}(A)$, the null space of $A$.

5. The following matrices $A$ and $B$ are row equivalent.
   \[
   A = \begin{bmatrix}
   1 & 2 & 1 & 11 & -3 \\
   2 & 4 & 1 & 15 & 2 \\
   1 & 2 & 0 & 4 & 5 \\
   3 & 6 & 1 & 19 & -2
   \end{bmatrix}, \quad B = \begin{bmatrix}
   1 & 2 & 0 & 4 & 0 \\
   0 & 0 & 1 & 7 & 0 \\
   0 & 0 & 0 & 0 & 1 \\
   0 & 0 & 0 & 0 & 0
   \end{bmatrix}.
   \]
   (a) Find a basis for $\text{Nul}(A)$, the null space of $A$.
   (b) Find a basis for $\text{Col}(A)$, the column space of $A$.

6. Let $V$ be an $n$-dimensional vector space. Suppose $S = \{v_1, \ldots, v_k\}$ is a subset of $V$ containing $k$ vectors with $k < n$. Explain why $S$ cannot span $V$. 
