1. Let \( A = \begin{bmatrix} 4 & -2 & -2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \).

(a) Find a basis for the eigenspace of \( A \) corresponding to \( \lambda = 1 \).
(b) Find a basis for the eigenspace of \( A \) corresponding to \( \lambda = 2 \).
(c) Find a basis for the eigenspace of \( A \) corresponding to \( \lambda = 3 \).

2. Suppose \( \lambda \) is an eigenvalue of an invertible matrix \( A \). Show that \( \frac{1}{\lambda} \) is an eigenvalue of \( A^{-1} \).

3. Let \( A \) be a \( n \times n \) matrix.

(a) Show that \( A \) and \( A^T \) have the same eigenvalues.
(b) Do \( A \) and \( A^T \) necessarily have the same eigenvectors? Explain.

4. Let \( A = \begin{bmatrix} -1 & -3 & 0 \\ 0 & 4 & 0 \\ 1 & 1 & 2 \end{bmatrix} \).

(a) Find the characteristic polynomial of \( A \).
(b) Find the eigenvalues of \( A \) and a basis for each of the corresponding eigenspaces of \( A \). (Hint: One of the eigenvalues is \( \lambda = 2 \).)

5. Let \( A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & h & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \).

Find the value(s) of \( h \) for which the dimension of the eigenspace of \( A \) corresponding to \( \lambda = 5 \) is 2.

6. Let \( A \) be a \( n \times n \) matrix with \( n \) distinct real eigenvalues \( \lambda_1, \ldots, \lambda_n \), so that
\[
\det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \cdots (\lambda_n - \lambda) .
\]

Explain why \( \det(A) \) is equal to the product of the \( n \) eigenvalues of \( A \).