Finish limits at ∞

$\lim_{x \to \infty} \frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are polynomials

$$\left(\text{i.e.} \quad \lim_{x \to \infty} \frac{x^2 + 2x + 3}{-x^3 - 1} \right)$$

if $\deg(f) > \deg(g)$ then $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty$ or $-\infty$ depending on the sign of the leading coefficients of f and g

if $\deg(f) < \deg(g)$ then $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$

if $\deg(f) = \deg(g)$, then $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \frac{\text{leading term of } f(x)}{\text{leading term of } g(x)}$

Rules

$$\frac{\text{nonzero}}{0} = \pm\infty$$

$$\frac{0}{\text{nonzero}} = 0$$

$$\frac{\infty}{\text{nonzero}} = \pm\infty$$

$$\frac{\text{nonzero}}{\pm\infty} = 0$$

Must Do Work Work

(indeterminate forms)

$$\frac{0}{0}$$

$$\frac{\pm\infty}{\pm\infty}$$

$$\frac{\pm\infty}{0} \text{ or } \frac{0}{\pm\infty}$$

$$\infty \pm \infty$$

2.4 Continuity

Def: A function f is <u>continuous at a number a</u> if

$$\lim_{x \to a} f(x) = f(a).$$

~~The~~ To check if $f(x)$ is continuous at a, must check three things:

1. a is in the domain of f (so $f(a)$ is defined)
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

If any of 1, 2, or 3 are not satisfied, then $f(x)$ is not continuous at a. We say f has a <u>discontinuity</u> (or is <u>discontinuous</u>) at a if this is the case.

Geometric meaning: $f(x)$ is continuous if the graph of f can be drawn without picking up your pencil.

The following types of functions are continuous at every number in their domain:

polynomials
root function
exponential functions
rational functions
trigonometric functions
logarithmic functions

Examples of discontinuities

① Where is $f(x) = \frac{x-1}{x^2-1}$ continuous? Where are its discontinuities?

$f(x)$ is a rational function, so $f(x)$ is continuous on its domain.

domain of $f(x)$: $x^2-1=0$
$(x-1)(x+1)=0$
$x=1$ or $x=-1$

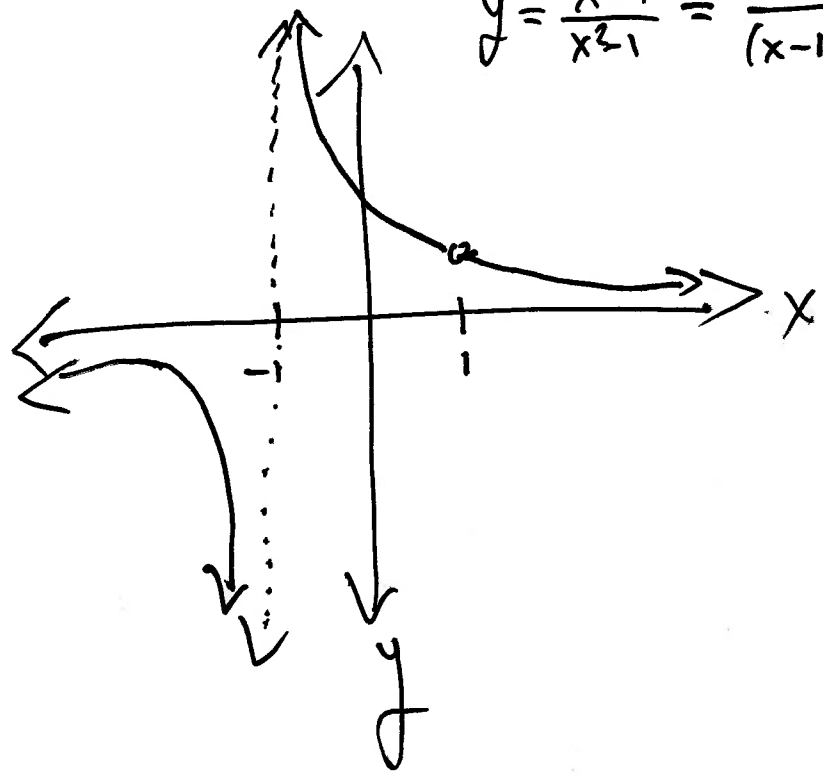
The domain of $f(x)$ is all x such that $x \neq 1$ and $x=-1$.

$f(x)$ is continuous at all x except $x=1$ and $x=-1$.

The discontinuities of $f(x)$ are at $x=1$ and $x=-1$.

$$y = \frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} \text{ "=" } \frac{1}{x+1}$$

<u>graph of f:</u>



(2) $f(x) = \begin{cases} 2x-1 & \text{if } x \neq 2 \\ 5 & \text{if } x = 2 \end{cases}$

where is $f(x)$ continuous?
where are discontinuities?

$\boxed{\text{2x-1 and 5 are polynomials so f(x) is continuous}}$

$2x-1$ is continuous everywhere since it is a polynomial.

Since $f(x)$ is $2x-1$ when $x \neq 2$, $f(x)$ is continuous when $x \neq 2$.

Since $x=2$ is a discrete point we cannot conclude $f(x)$ is continuous at $x=2$ because 5 is a polynomial.

$f(x)$ is continuous everywhere except possibly at $x=2$.

<u>At $x=2$</u>

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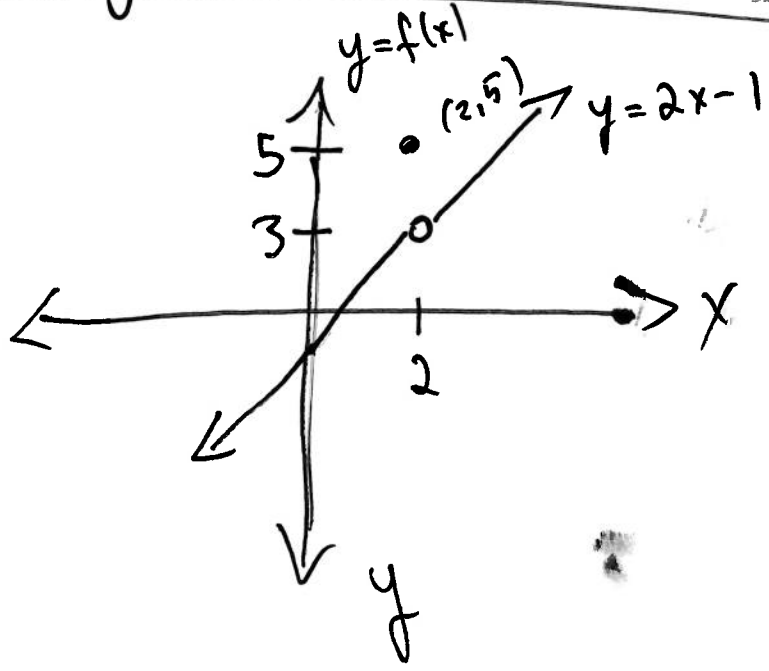
$$\lim_{x\to 2} f(x) \xlongequal[f(x)=2x-1]{\text{because when } x\neq 2} \lim_{x\to 2} 2x-1 = 2\cdot 2-1 = 3$$

$$f(2) = 5$$

$\lim_{x\to 2} f(x) = 3 \neq 5 = f(2)$ so $f(x)$ is not continuous at $x=2$.

Discontinuities of f are at $x=2$. $f(x)$ is continuous everywhere else.



(3) $f(x) = \begin{cases} 2x-1 & \text{if } x < 2 \\ x^2 & \text{if } x \geq 2 \end{cases}$

Where is f continuous where are discontinuities?

The only place f(x) is possibly discontinuis is at $x=2$

At $x=2$

$\lim_{x \to 2} f(x) =$ to determine the limit we calculate the left and right limits

$f(2) = 2^2 = 4$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2 = 2^2 = 4$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 2x-1 = 2 \cdot 2 - 1 = 3$

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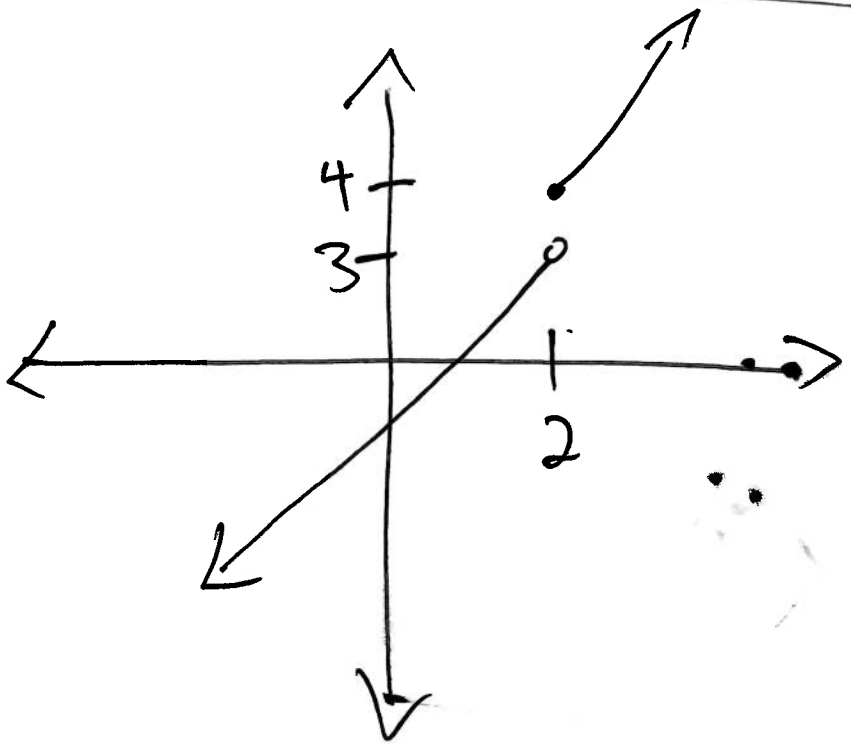
$\lim_{x \to 2} f(x)$ does not exist because

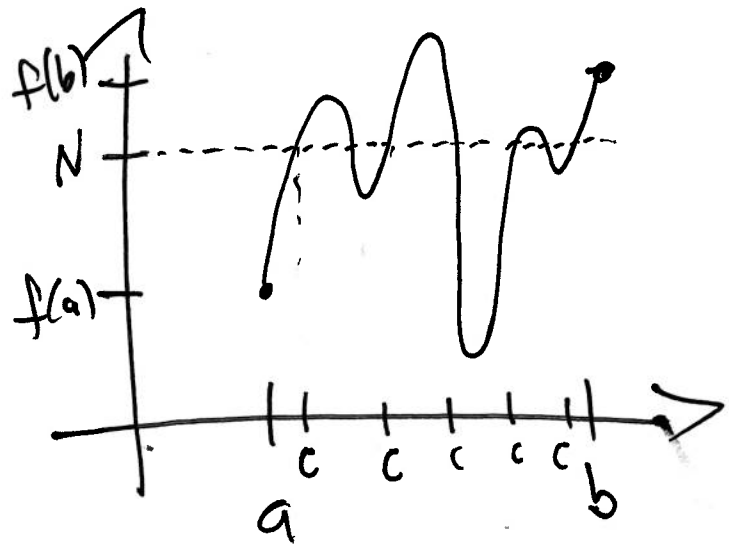
$$\lim_{x \to 2^-} f(x) = 3 \neq 4 = \lim_{x \to 2^+} f(x).$$

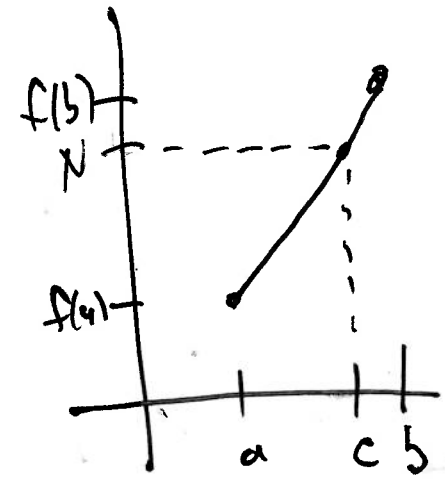
$\boxed{\text{Therefore } f(x) \text{ is not continuous at } x = 2.}$



Intermediate Value Theorem (IVT)

Suppose f is continuous on the closed interval $[a,b]$, and let N be any number between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$. Then there exists a c in (a,b) such that $f(c) = N$.





<u>Example</u> (Using IVT to prove equations have solutions.)

(1) Show that $e^x = 3 - 2x$ has a solution for x.

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Use IVT:

- Move everything to one side

$$0 = 3 - 2x - e^x$$

- set $f(x) = 3 - 2x - e^x$
- Now find x such that $f(x) = 0$.
  - find a such that $f(a) < 0$
    b " " $f(b) > 0$

    then there exists by IVT a c such that $f(c) = 0$. $(N = 0)$

Try $a = 0$, $b = 1$

$$f(0) = 3 - 2\cdot 0 - e^0 = 3 - 0 - 1 = 2 > 0$$
$$f(1) = 3 - 2\cdot 1 - e^1 = 3 - 1 - 2.\text{something} < 0$$

By IVT there exists a c between 0 and 1 such that

$$f(c) = 0, \quad 3 - 2c - e^c = 0$$