

③ ~~f(x)~~ is not differentiable at ~~(x₀, f(x₀))~~ if the graph $y = f(x)$ is not smooth at $(x_0, f(x_0))$ or has a sharp edge or has a corner.

EX: $f(x) = |x|$

Graph $y = |x|$ has a sharp edge / corner at $(0,0)$.

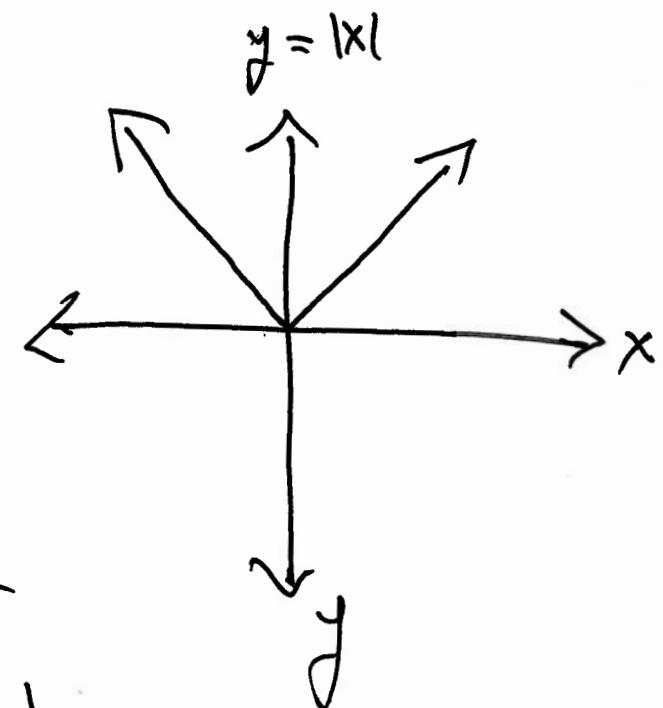
Show that $f(x) = |x|$ is not differentiable at $x=0$.

To do this, we need to show that

$$f'(0) = \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h}$$

does not exist.

$$\frac{f(0+h) - f(0)}{h} = \frac{|0+h| - |0|}{h} = \frac{|h|}{h}$$



$$\lim_{h \rightarrow 0} \frac{|h|}{h}, \quad |h| = \begin{cases} h & \text{if } h \geq 0 \\ -h & \text{if } h < 0 \end{cases}$$

Since $|h|=h$ if $h \geq 0$,

$$\lim_{h \rightarrow 0^+} \frac{|h|}{h} = \lim_{h \rightarrow 0^+} \frac{h}{h} = \lim_{h \rightarrow 0^+} 1 = 1$$

Since $|h|=-h$ if $h < 0$,

$$\lim_{h \rightarrow 0^-} \frac{|h|}{h} = \lim_{h \rightarrow 0^-} \frac{-h}{h} = \lim_{h \rightarrow 0^-} -1 = -1$$

Then $\lim_{h \rightarrow 0} \frac{|h|}{h}$ does not exist because the left and right limits are different.

Therefore $f(x)=|x|$ is not differentiable at $x=0$.

Other notations for derivative

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}, \quad f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$$

$$h = x - a, \quad x = a + h$$

$$\text{as } h \rightarrow 0, x \rightarrow a$$

$y = f(x)$, $f'(x) = \text{derivative of } f \text{ with respect to } x$

$$\frac{df}{dx} = "$$

$$\frac{dy}{dx} = "$$

$$y' = "$$

$$\frac{d}{dx}(f(x)) = "$$

Higher Derivatives

$f''(x)$ = 2nd derivative of $f(x)$ = derivative of $f'(x)$

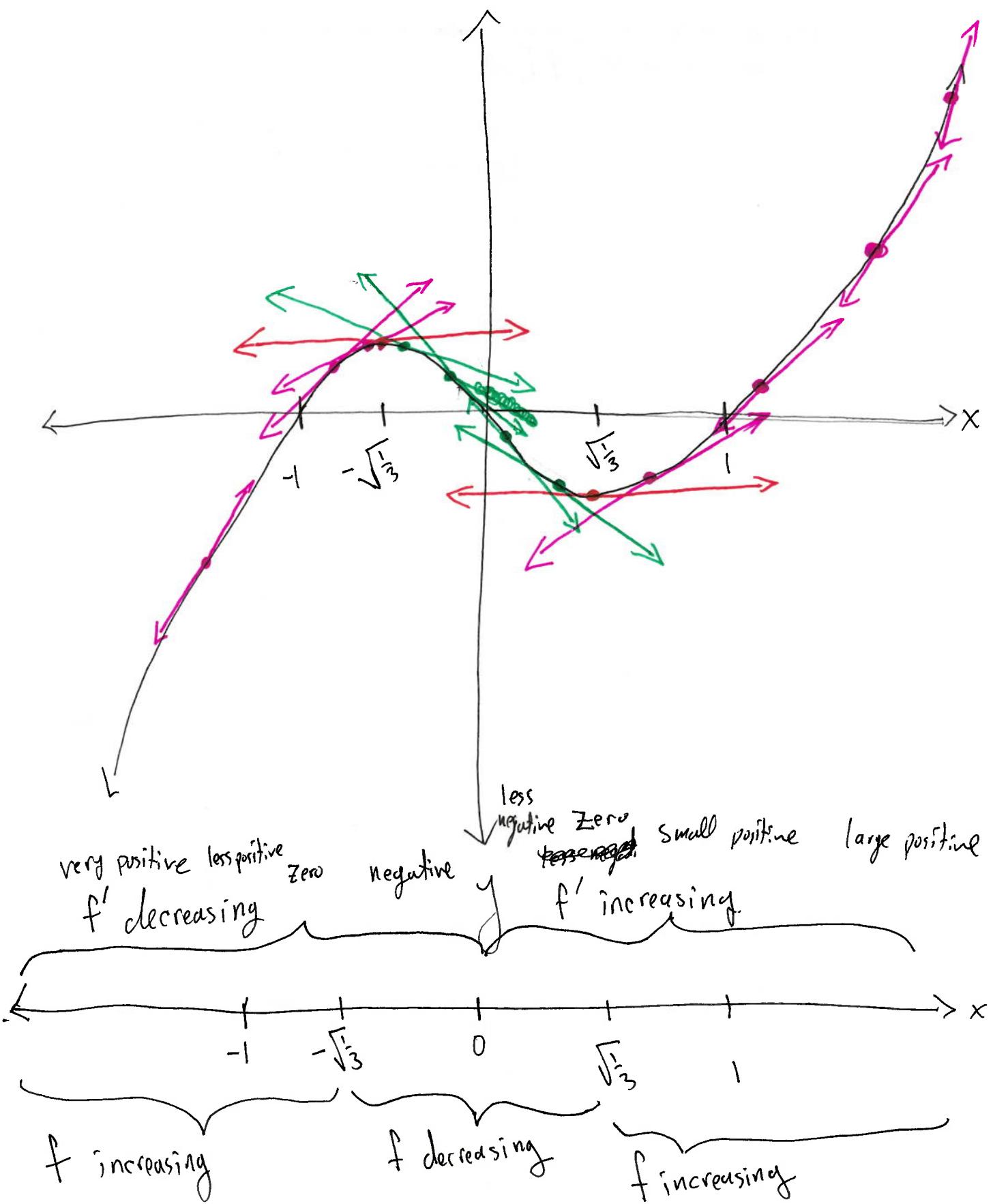
$f^{(5)}(x)$ = 5th derivative of $f(x)$

$\frac{d^2 f}{dx^2}$ = 2nd derivative of $f(x)$

~~$\frac{d^2 f}{dx^2} = 2\cos \theta$~~ $\frac{d^2 y}{dx^2} = 2\text{nd derivative of } f(x)$

2.8 What does f' say about f

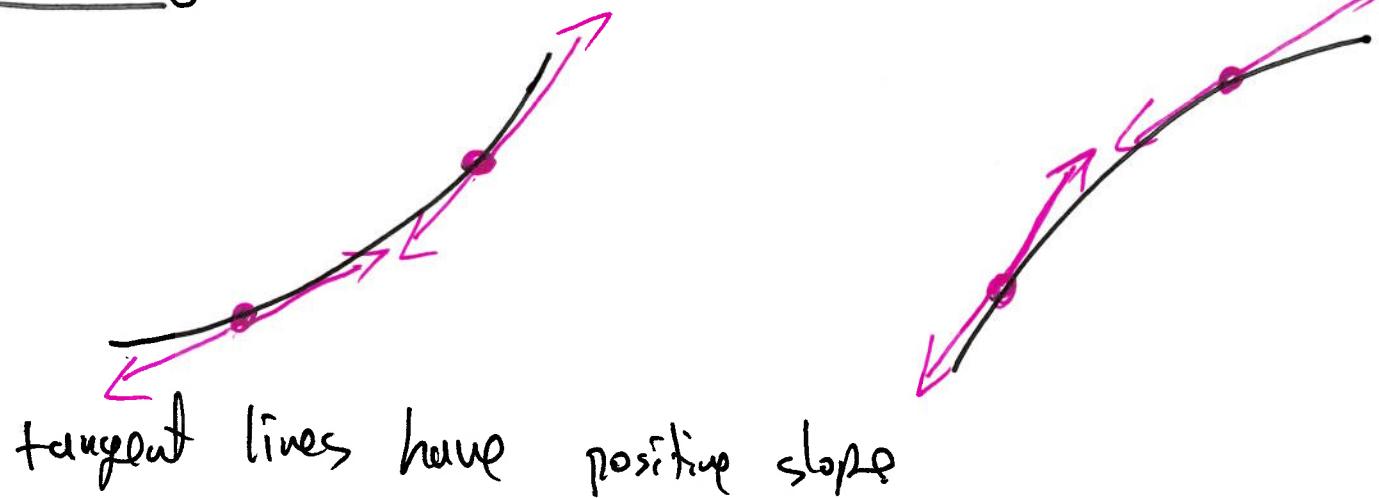
$$y = x^3 - x = x(x-1)(x+1)$$



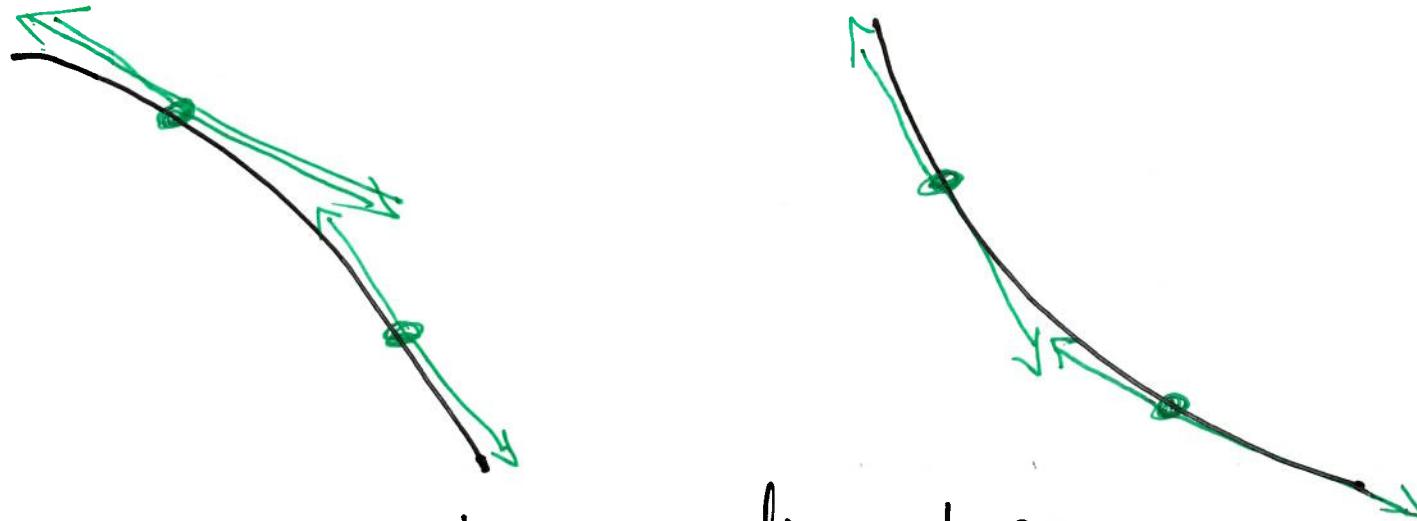
$f'(x_0)$ is the rate of change of $f(x)$ at x_0

- If $f'(x_0) \geq 0$ on an interval, then $f(x)$ is increasing on the interval.
- If $f'(x_0) < 0$ on an interval, then $f(x)$ is decreasing on that interval.

Increasing



Decreasing

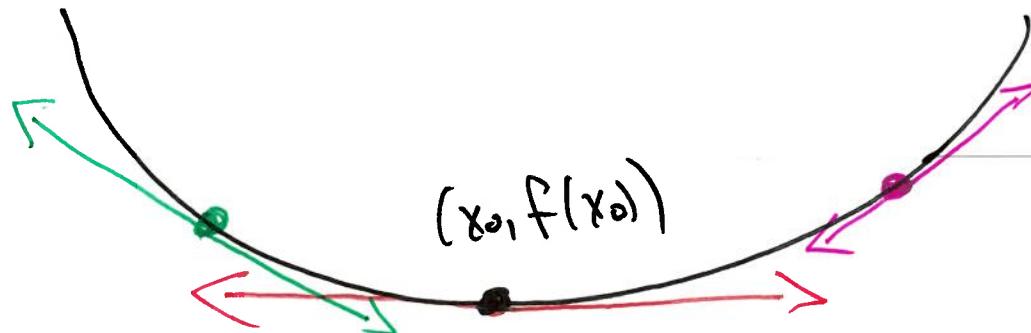


tangent lines have negative slope.

• x_0 is a local minimum of $f(x)$ if

(1) $f'(x_0) = 0$

(2) f changes from decreasing to increasing at x_0
($f'(x)$ changes from negative to positive at x_0)

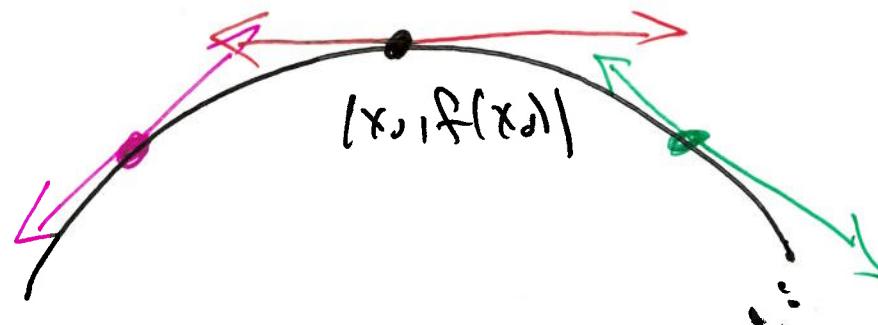


local min.

- x_0 is a local maximum of $f(x)$ if

$$(1) \quad f'(x_0) = 0$$

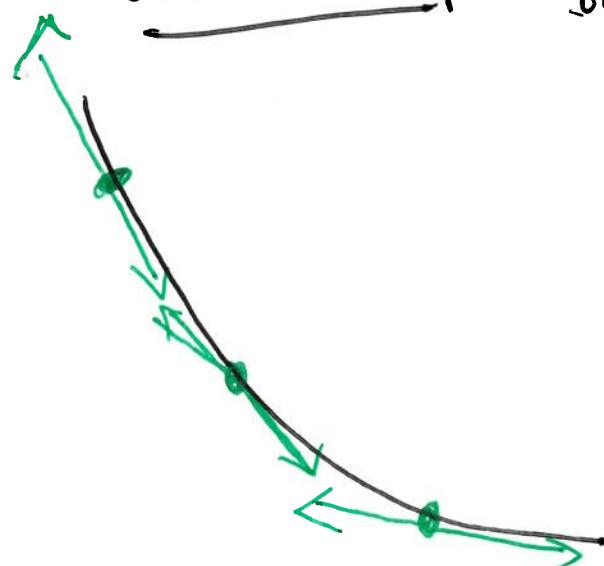
- $$(2) \quad f \text{ changes from increasing to decreasing at } x_0$$



What does f'' say about f ?

- If $f''(x) > 0$, $f'(x)$ is increasing.
- If $f''(x) < 0$, $f'(x)$ is decreasing

If $f''(x) > 0$ on an interval, then $f(x)$ is said to be concave up on the interval.



f is concave up

f' and decreasing

f' is becoming less negative (increasing)

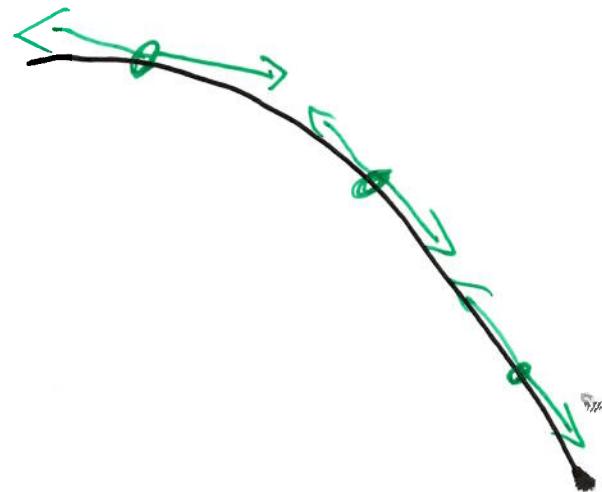


f is concave up

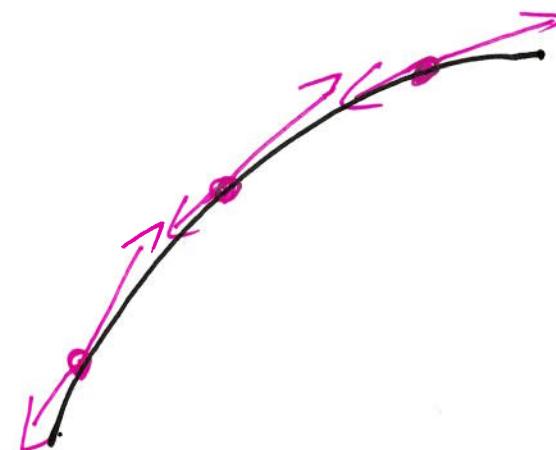
and increasing

f' is becoming more positive
(increasing)

If $f''(x) < 0$ on an interval, then $f(x)$ is said to be concave down on the interval.



f is concave down
and decreasing
 f' is decreasing
slopes become more negative



f is concave up
and increasing
 f' is decreasing
slopes become less positive

x_0 is a point of inflection if
(p.o.i.)

$$\textcircled{D} f''(x_0) = 0$$

② The concavity of f changes at x_0 .

