

Midterm 2

ave. 60%

std. dev. 17%

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$e = \text{Euler's number} = 2.\underline{\text{something}}$

$$f(x) = \ln(x) \quad [f'(x) = e^x],$$

$$f'(x) = \frac{1}{x}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{1}{x} = \lim_{h \rightarrow 0} \frac{\ln(x+h) - \ln(x)}{h}$$

$$\left[\frac{\ln(x+h) - \ln(x)}{h} \right] = \frac{\ln\left(\frac{x+h}{x}\right)}{h} = \frac{1}{h} \ln\left(\frac{x+h}{x}\right) = \ln\left(\left(\frac{x+h}{x}\right)^{1/h}\right)$$

$\hookrightarrow = \ln\left(\left(1 + \frac{h}{x}\right)^{1/h}\right)$

$$\frac{1}{x} = \lim_{h \rightarrow 0} \ln\left(\left(1 + \frac{h}{x}\right)^{1/h}\right)$$

$$e^{1/x} = e^{\lim_{h \rightarrow 0} \ln\left(\left(1 + \frac{h}{x}\right)^{1/h}\right)} = \lim_{h \rightarrow 0} e^{\ln\left(\left(1 + \frac{h}{x}\right)^{1/h}\right)}$$

$$= \lim_{h \rightarrow 0} \left(1 + \frac{h}{x}\right)^{1/h}$$

✓

$$e^{\frac{1}{x}} = \lim_{h \rightarrow 0} \left(1 + \frac{h}{x}\right)^{1/h}$$

$$\begin{aligned} n &= \frac{1}{h}, \quad hn = 1 \\ h &= \frac{1}{n} \end{aligned}$$

$$e^{\frac{1}{x}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{nx}\right)^n$$

$\Rightarrow h \rightarrow 0^+, n \rightarrow \infty$

plug in $x=1$ get

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$\text{substitute } z = \frac{1}{x} \quad x = \frac{1}{z}$$

$$e^z = \lim_{n \rightarrow \infty} \left(1 + \frac{z}{n}\right)^n$$

4.2 Maximum and Minimum Values

Optimization

- What is the shape of a can that minimizes manufacturing costs?
- What is the radius of a contracted wind pipe that expels air most rapidly during a cough?

Local and absolute mins/maxs

D = domain of a function f

c = number in D

$f(c)$ is the absolute maximum of f on D if $f(c) \geq f(x)$ for all x in D

$f(c)$ " " " absolute minimum " "

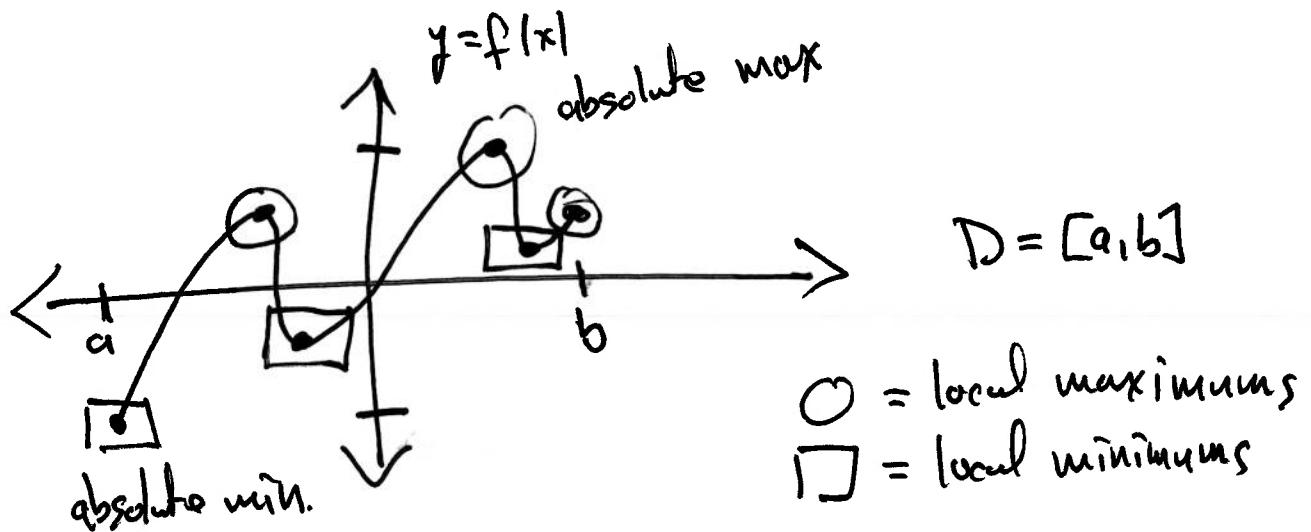
$f(c)$ " " " local maximum " "

$f(c)$ " " " local minimum " "

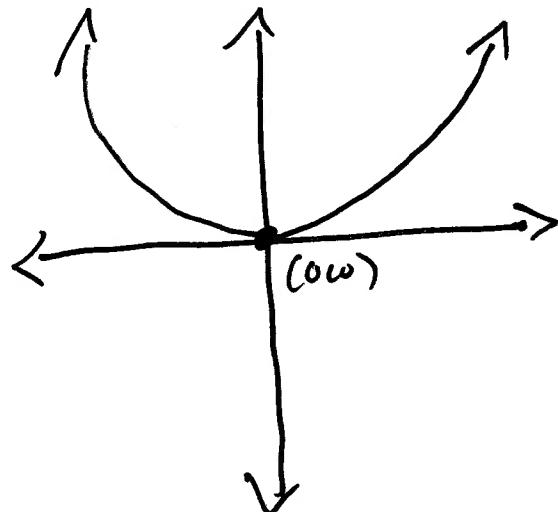
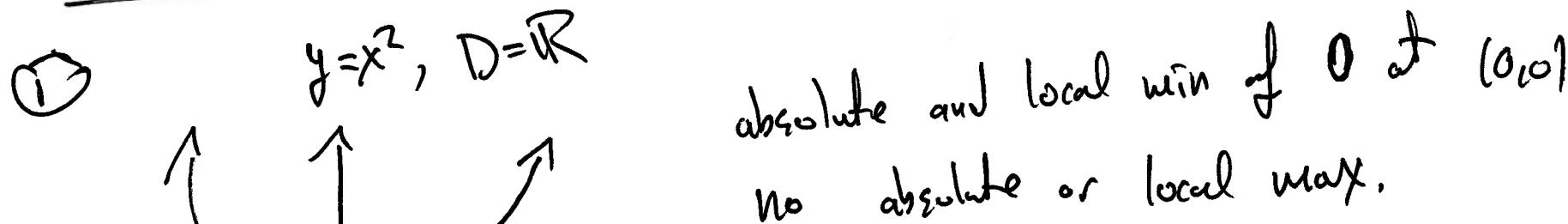
" $f(c) \geq f(x)$ near c

" $f(c) \leq f(x)$ " "

Picture Example



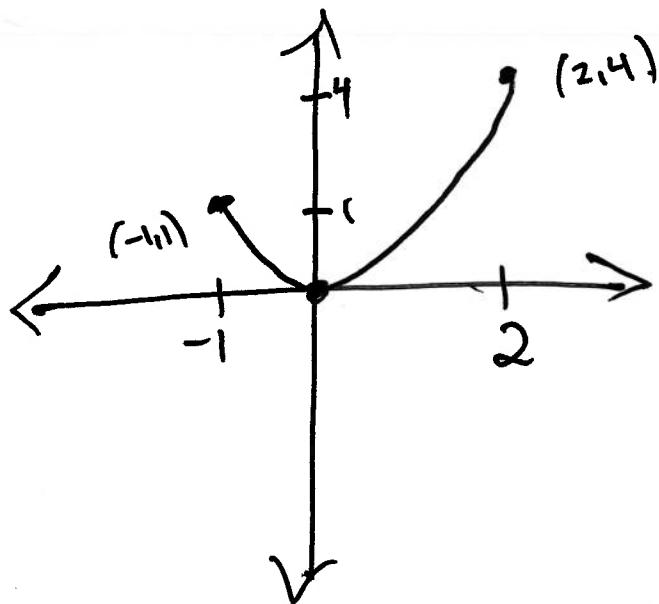
More pictures



2

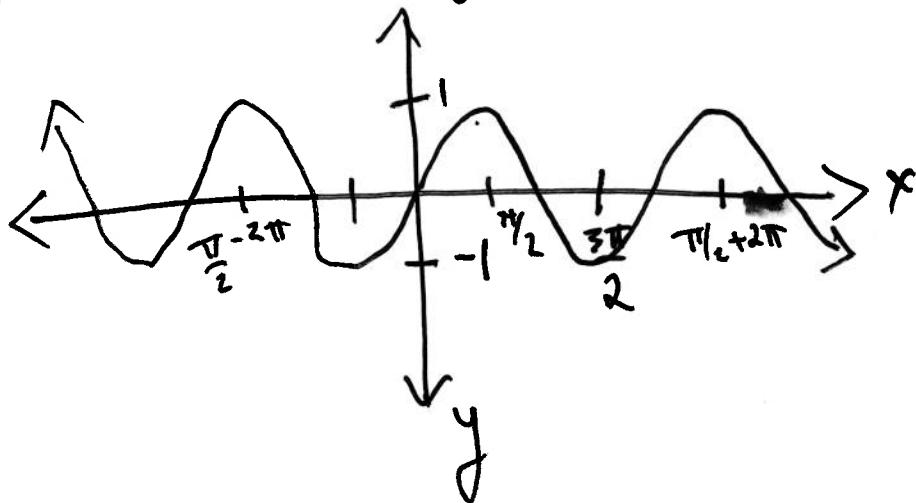
(1.5)

$$y = x^2, D = [-1, 2]$$



(2)

$$y = \sin(x), D = \mathbb{R}$$



absolute and local min of 0
at $(0, 0)$.

local max of 1 at $(-\frac{\pi}{2}, -1)$
absolute and local max of 4.
at $(2, 4)$

absolute and local max

of 1 at $x = \frac{\pi}{2} + 2\pi k$

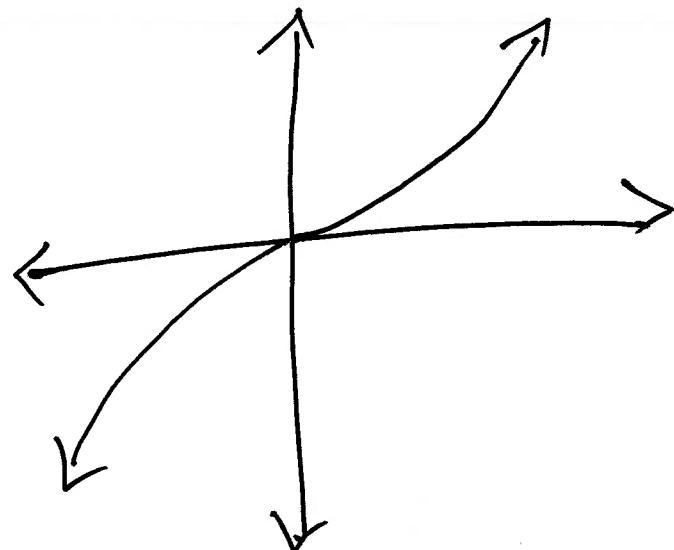
where k is a whole number

absolute and local min

of -1 at $x = \frac{3\pi}{2} + 2\pi k$

where k is a whole number

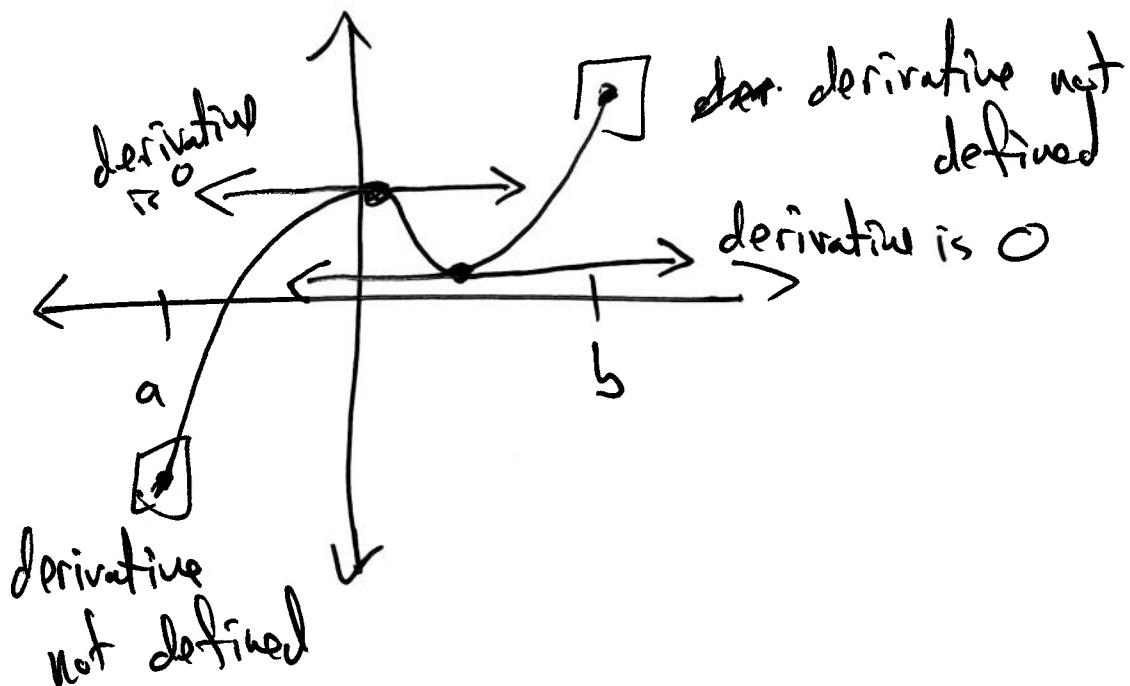
(3) $y = x^3, D = \mathbb{R}$



No absolute or local
maxes or mins.

Extreme Value Thm: If f is continuous on a closed interval $[a, b]$
then f attains an absolute max value $f(c)$ and an
absolute min value $f(d)$ on for some numbers c and
 d in $[a, b]$.

Fermat's Thm: If f has a local max or min at c and if $f'(c)$ is defined, then $f'(c) = 0$.



Def: A critical number of f is a number $c \in D$ where either $f'(c) = 0$ or $f'(c)$ is not defined.

If f has a local max or min at c , then c is a critical number for f .

Closed Interval Method

To find the absolute max and min values of a continuous function f on $D = [a, b]$:

① Find critical numbers of f in (a, b)

1. Find the y -values of f at critical number.

2. Find the y -values of f at the ~~as~~ endpoints of interval $(x=a, x=b)$.

3. The largest of the y -values from steps 1 and 2 is the absolute maximum. The smallest is the absolute minimum.

Examples

① $f(x) = 2x^3 - 3x^2 - 12x + 1$, $D = [-2, 3]$

Find absolute max and min of f on D .

Step 0

$$f'(x) = 6x^2 - 6x - 12$$

$$0 = 6x^2 - 6x - 12$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$x=2 \text{ or } x=-1$$

critical numbers are

$$x=2, x=-1$$

Step 1

$$f(2) = ?, \quad f(-1) = ?$$

$$f(2) = 2 \cdot (2)^3 - 3 \cdot (2)^2 - 12 \cdot 2 + 1$$

$$= 2 \cdot 8 - 3 \cdot 4 - 24 + 1$$

$$= 16 - 12 - 23 = 4 - 23 = -19$$

$$f(2) = -19$$

$$f(-1) = 8$$

$$f(-1) = 2 \cdot (-1)^3 - 3 \cdot (-1)^2 - 12 \cdot (-1) + 1$$

$$= -2 - 3 + 12 + 1 = -5 + 13 = 8$$

Step 2 $a = -2, b = 3, \quad f(-2) = ?, \quad f(3) = ?$

$$f(-2) = 2(-2)^3 - 3(-2)^2 - 12 \cdot (-2) + 1$$

$$f(-2) = -3$$

$$= 2 \cdot (-8) - 3 \cdot 4 + 24 + 1$$

$$= -16 - 12 + 25 = -28 + 25 = -3$$

$$f(3) = 2 \cdot 3^3 - 3 \cdot 3^2 - 12 \cdot 3 + 1$$

$$f(3) = -8$$

$$= 2 \cdot 27 - 3 \cdot 9 - 36 + 1$$

$$= 54 - 27 - 35 = 27 - 35 = -8$$

Step 3 $f(2) = -19$, $\underbrace{f(-1) = 8}_{\text{abs min.}}$, $\underbrace{f(-2) = -3}_{\text{abs max}}$, $f(3) = -8$

f has an absolute max of 8 at $x = -1$
" " " " min of -19 at $x = 2$