No in person final next week.
Plan to do online final...
If online final new grade breakdown:

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>26%</td>
</tr>
<tr>
<td>M2</td>
<td>26%</td>
</tr>
<tr>
<td>Final</td>
<td>28%</td>
</tr>
<tr>
<td>HW</td>
<td>20%</td>
</tr>
</tbody>
</table>

Calculating your grade:

\[
\frac{\text{score on M1}}{50} \times 0.26 + \frac{\text{score on M2}}{42} \times 0.26 + \frac{\sum \text{HW scores}}{800} \times 0.20 + \frac{\text{final score}}{100} \times 0.28
\]

\[= 0.72\]
L'Hopital's Rule (use it for \( \frac{0}{0} \) or \( \frac{\infty}{\infty} \))

Indeterminate form \( 0 \cdot \infty \)

Example: \( \lim_{x \to 0^+} x \ln(x) \)

\[
\lim_{x \to 0^+} x \ln(x) = \lim_{x \to 0^+} \frac{\ln(x)}{\frac{1}{x}} \quad \text{plug in } 0 \quad \frac{\ln(0)}{0} = \frac{-\infty}{\pm \infty}
\]

\[
\lim_{x \to 0^+} \frac{f(x)}{g(x)} = \lim_{x \to 0^+} \frac{f'(x)}{g'(x)}
\]

\[
f(x) = \ln(x), \quad f'(x) = \frac{1}{x}
\]

\[
g(x) = x, \quad g'(x) = -x^2 = \frac{-1}{x^2}
\]

\[
\lim_{x \to 0^+} x \ln(x) = \lim_{x \to 0^+} \frac{\ln(x)}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{1}{x} \cdot x^2 = \lim_{x \to 0^+} \frac{1}{x} \cdot \frac{x^2}{1}
\]
4.6 Optimization Problems

\[ \lim_{x \to 0^+} \frac{x}{1 - x} = \infty \]

Q - function to be maximized or minimized (will be a function of 2 variables)

C: constraint equation involving 2 variables of Q.

Solve for a variable in constraint equation, then plug that into Q to eliminate one of the variables in Q. Then do calculus.
Examples

1. Find dimensions of a rectangle with perimeter 100m. whose area is as large as possible.

\[
\begin{align*}
x & \quad y \\
\quad x & \quad y
\end{align*}
\]

Area = $xy$

Perimeter = $2x + 2y$

\[
Q(x, y) = xy \quad 0 \leq x \leq 50, \quad 0 \leq y \leq 50
\]

C: $2x + 2y = 100 \quad \text{solve for } y$

\[
\begin{align*}
y & = \frac{100 - 2x}{2} \\
y & = 50 - x
\end{align*}
\]

\[
Q(x, 50-x) = x(50-x)
\]

\[
Q(x) = 50x - x^2
\]

\[
\begin{cases}
f(x) = 50x - x^2 \\
x \in [0, 50]
\end{cases}
\]

\[
\text{plug into } Q(x, y)
\]
Find max value (global max) of \( A(x) = 50x - x^2 \) on \([0, 50]\).

\[ A'(x) = 50 - 2x \]

\[ 0 = 50 - 2x \]

\[ 2x = 50 \]

\[ x = 25 \quad \text{[x-value that gives largest area]} \]

\[ y = 50 - x, \quad y = 50 - 25 = 25 \]

The rectangle with perimeter 100m and maximal area has dimensions 25m x 25m.
2) Find two positive numbers whose product is 100, and whose sum is minimal.

\[ \text{minimize the sum} \]

\[ xy \text{ two numbers, } x > 0, \ y > 0 \]

\[ Q(xy) = x + y \text{ subject to} \]

\[ C: \ xy = 100 \text{ constraint} \]

\[ \text{solve for } y \]

\[ y = \frac{100}{x} \text{ plug into } Q(xy) \text{ to eliminate } y \]

\[ Q \left( x, \frac{100}{x} \right) = x + \frac{100}{x} \]

\[ Q \left( x \right) = x + \frac{100}{x} \text{ minimize this function on } (0, \infty) \]
\( Q(x) = x + 100x^2 \)
\( Q'(x) = 1 - 100x^2 \)
\( Q''(x) = 1 - \frac{100}{x^2} \)
\( 0 = 1 - \frac{100}{x^2} \)
\( \frac{160}{x^2} = 1 \)
\( 160 = x^2 \)
\( \sqrt{160} = x \)
\( 10 = x \)

2nd der. test to check this is minimum.

\( Q''(x) = 0 + 200x^{-3} \)
\( Q''(10) = \frac{200}{1000} > 0 \)

\( x = 10 \) is minimum by 2nd derivative test.

The two numbers whose product is 100
and whose sum is minimal is 10 and 10.
\[ xy = 100 \]
\[ x + y \text{ minimal} \]

\[ x = 100, \quad y = 1 \quad \land \quad x + y = 101 \]

\[ x = 50, \quad y = 2 \quad \land \quad x + y = 52 \]

\[ x = 25, \quad y = 4 \quad \land \quad x + y = 29 \]

\[ x = 1, \quad y = \frac{100}{x} \quad \land \quad x + y = \frac{100}{x} + \frac{100}{1} = \frac{221}{x} \]

\[ x = 1 \quad y = 10 \quad \land \quad x + y = 20 \]

\[ \text{minimal sum} \]
A cylindrical can is to be made to hold 1000 cm³ of oil. Find dimensions of the can that will minimize the cost of material to manufacture the can.

Material to manufacture = surface area of cylinder (including top & bottom)

Material to hold the amount can hold = volume of the can

Volume = (area of base)height = \( \pi r^2 h \)

Surface area = \( 2\pi rh + \pi r^2 + \pi r^2 \)