No in person final next week.

Plan to do online final...

If online final new grade breakdown:

M1	26%
M2	26%
Final	28%
HW	20%

curve M2 so it is no out of 42 instead of 50

Calculating your grade

$$\underbrace{\frac{\text{score on M1}}{50}\cdot 26 + \frac{\text{score on M2}}{42}\cdot 26 + \frac{\text{sum HW scores}}{800}\cdot 20}_{0\ -\ 72} + \frac{\text{final score}}{100}\cdot 28$$

L'Hospital's Rule (use it for $\frac{0}{0}$ or $\frac{\infty}{\infty}$)

$\ln(0) = y$, $e^y = 0$

$\lim_{x\to 0^+} \ln(x) = -\infty$, $\lim_{y\to -\infty} e^y = 0$

indeterminate form $0 \cdot \pm\infty$

Example $\lim_{x\to 0^+} x\ln(x)$ plug in 0 $\quad 0\cdot\ln(0) = 0\cdot -\infty$ indeterminate form.

$$\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{\frac{1}{x}}$$

plug in 0 $\quad \frac{\ln(0)}{\frac{1}{0}} = \frac{-\infty}{\pm\infty}$

l'H $\quad \lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$

$f(x) = \ln(x)$, $\quad f'(x) = \frac{1}{x}$

$g(x) = \frac{1}{x} = x^{-1}$ $\quad g'(x) = -x^{-2} = \frac{-1}{x^2}$

$$\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{\frac{1}{x}} \overset{l'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{1}{x}\cdot\frac{x^2}{-1}$$

$$\lim_{x \to 0^+} \frac{x}{-1} = \frac{0}{-1} = \boxed{0}$$

4.6 Optimization Problems

Q – Function to be maximized or minimized
(will be a function of 2 variables)

C : constraint equation involving 2 variables of Q.

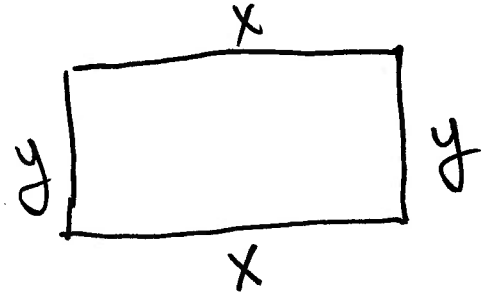
Solve for a variable in constraint equation, then plug that into Q to eliminate one of the variables in Q.

Then do calculus.

Examples

(1) Find dimensions of a rectangle with $\boxed{\text{perimeter } 100\text{m}}$ (constraint) whose area is as large as possible.

maximize area



$$\text{Area} = xy$$
$$\text{Perimeter} = 2x + 2y$$

$$Q(x,y) = xy \qquad 0 \le x \le 50,\ 0 \le y \le 50$$

$$C: \quad 2x + 2y = 100 \longleftarrow \text{solve for } y$$

$$2y = 100 - 2x$$

$$y = \frac{100}{2} - \frac{2x}{2}$$

$$y = 50 - x \longleftarrow \text{plug into } Q(x,y)$$

$$Q(x, 50-x) = x(50-x)$$

$$Q(x) = 50x - x^2$$

$$\left(f(x) = 50x - x^2 \quad x \text{ is in } [0, 50] \right)$$

Find max value (global max) of $Q(x) = 50x - x^2$ (on $[0, 50]$).

$Q'(x) = 50 - 2x$

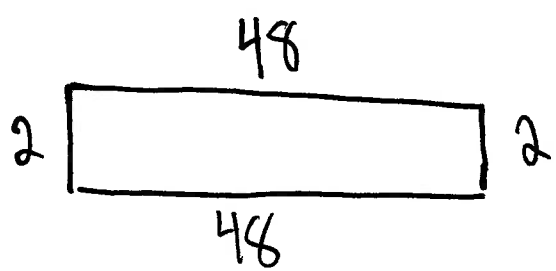
$0 = 50 - 2x$

$2x = 50$

$x = 25$ ← x-value that gives largest area.

$y = 50 - x$, $y = 50 - 25 = 25$

$\boxed{\text{The rectangle with perimeter 100m and maximal area has dimensions } 25\text{m} \times 25\text{m}}$



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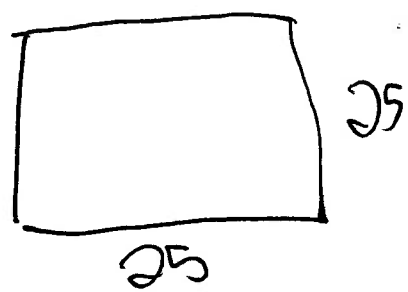
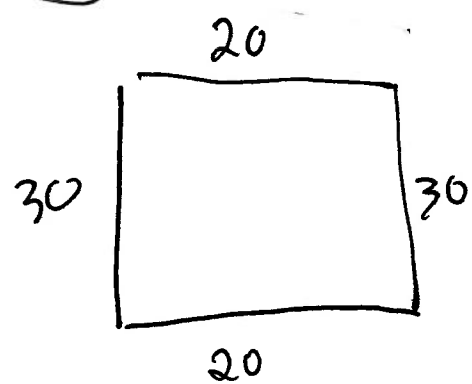


$625 = 25 \cdot 25$



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② Find two positive numbers whose $\boxed{\text{product is } 100}$ constraint.
and whose <u>sum is minimal.</u>

↖ minimize the sum

x, y two numbers, $x>0$, $y>0$

$Q(x,y) = x + y$

$C: \quad xy = 100$ constraint

↖ solve for y

$y = \frac{100}{x}$ ← plug into $Q(x,y)$ to eliminate y

$Q\left(x, \frac{100}{x}\right) = x + \frac{100}{x}$

$Q(x) = x + \frac{100}{x}$ ← minimize this function on $(0, \infty)$

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$$Q(x) = x + 100x^{-1}$$

$$Q'(x) = 1 - 100x^{-2}$$

$$Q'(x) = 1 - \frac{100}{x^2}$$

$$0 = 1 - \frac{100}{x^2}$$

$$\frac{100}{x^2} = 1$$

$$100 = x^2$$

$$\sqrt{100} = x$$

$$10 = x$$

2nd der. test to check this is minimum.

$$Q''(x) = 0 + 200x^{-3}$$

$$Q''(x) = \frac{200}{x^3}$$

$$Q''(10) = \frac{200}{10^3} > 0$$

$x = 10$ is minimum by 2nd derivative test.

$$y = \frac{100}{x}, \quad y = \frac{100}{10} = 10$$

The two numbers whose product is 100 and whose sum is minimal is 10 and 10.

$xy = 100$

$x + y$ minimal

$x = 100, \quad y = 1 \qquad x + y = 101$

$x = 50, \quad y = 2 \qquad x + y = 52$

$x = 25 \quad y = 4 \qquad x + y = 29$

$x = 11 \quad y = \frac{100}{11} \qquad x + y = \frac{121}{11} + \frac{100}{11} = \frac{221}{11}$

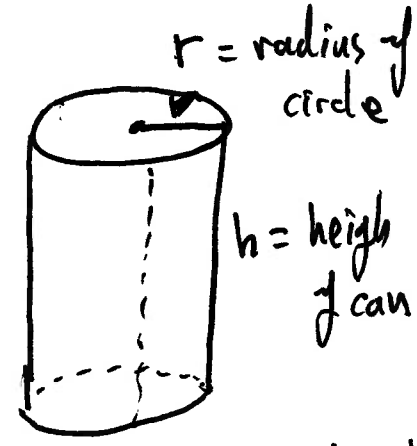
$x = 10, \quad y = 10 \qquad x + y = 20$

minimal sum

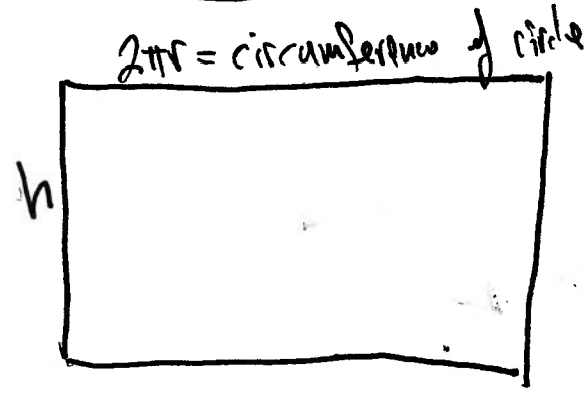
Constraint

(3) A cylindrical can is to be $\boxed{\text{made to hold } 1000\ cm^3}$ of oil. Find dimensions of can that will minimize the cost of material to manufacture the can.

material to manufacture can = surface area of cylinder (including top & bottom)



~~made to hold~~ amount can can hold = volume of the can



volume = (area of base) height = $\pi r^2 h$

Surface area = $2\pi r h + \pi r^2 + \pi r^2$

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