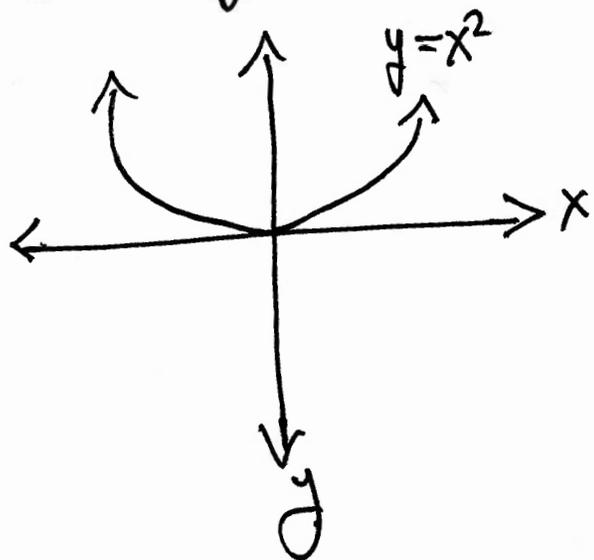


1.3 New Functions from Old Functions

Translating Functions

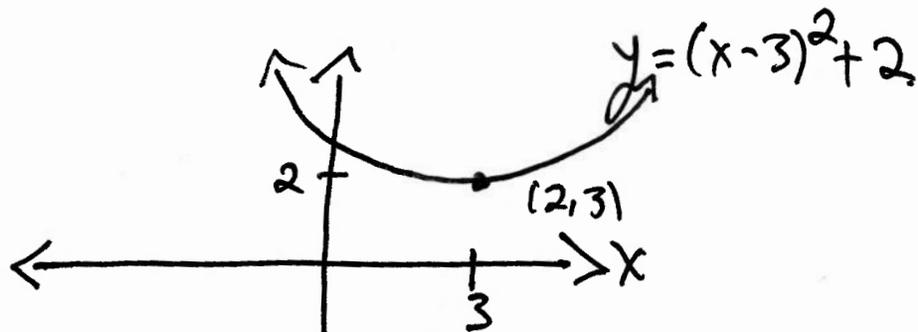


$$f(x) = x^2$$

$$g(x) = f(x-3) + 2$$

$$g(x) = (x-3)^2 + 2$$

\mathbb{R}



same shape as $y = x^2$
but shifted 3
to right and
up 2.

how you get $g(x) = (x-3)^2 + 2$
from $f(x) = x^2$

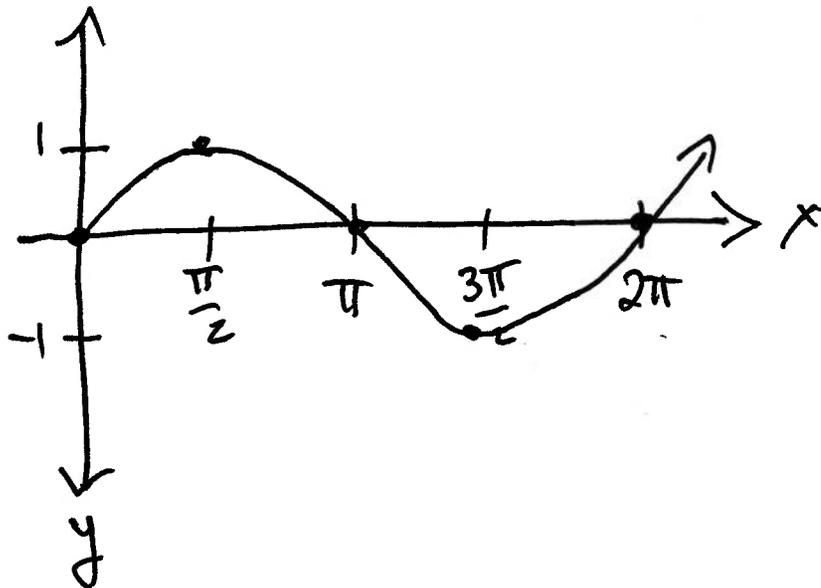
Rules for shifts: Let $c > 0$ be a constant

$y = f(x) + c$ shifts the graph $y = f(x)$ ~~up~~ c units up
 $y = f(x) - c$ shifts " " c units down

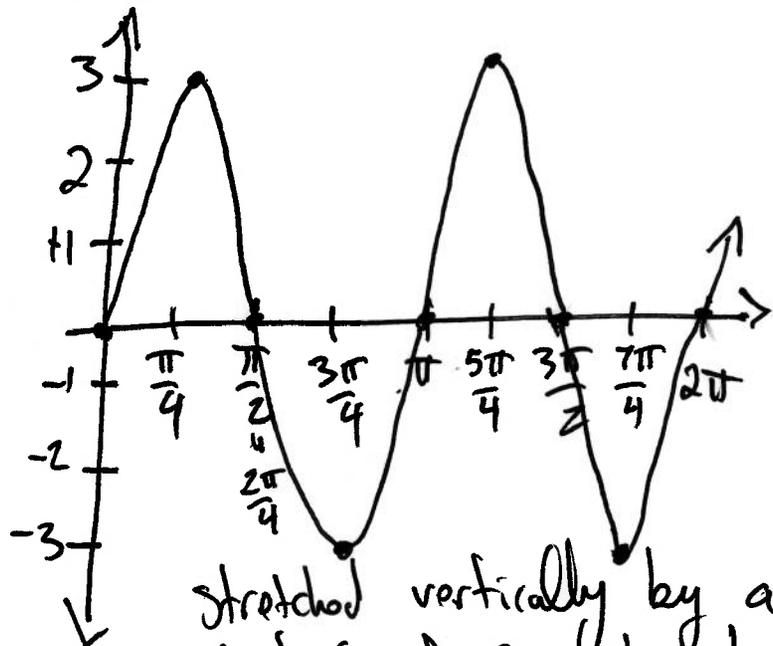
$y = f(x - c)$ " " c units to right
 $y = f(x + c)$ " " c units to left

Stretching Functions

$$y = \sin(x)$$



$$y = 3\sin(2x)$$



stretched vertically by a factor of 3, shrunk horizontally by a factor of $\frac{2}{2}$

$$f(x) = \sin(x)$$

$$g(x) = 3f(2x) \leftarrow \text{how to obtain } g(x) = 3\sin(2x)$$

$$g(x) = 3\sin(2x)$$

$$\text{from } f(x) = \sin(x)$$

Rules for stretching/shrinking: $c > 1$

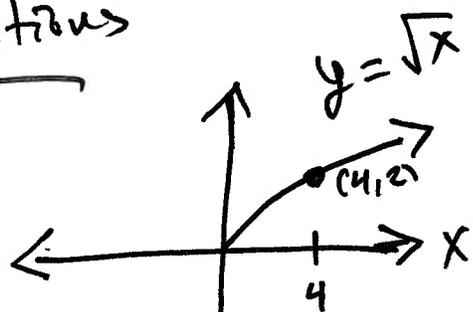
$y = cf(x)$ stretches the graph of $f(x)$ vertically by a factor of c

$y = \frac{1}{c}f(x)$ shrinks " " vertically by a factor of c

$y = f(cx)$ shrinks " " horizontally by a factor of c

$y = f(\frac{1}{c}x)$ stretches " " horizontally by a factor of c

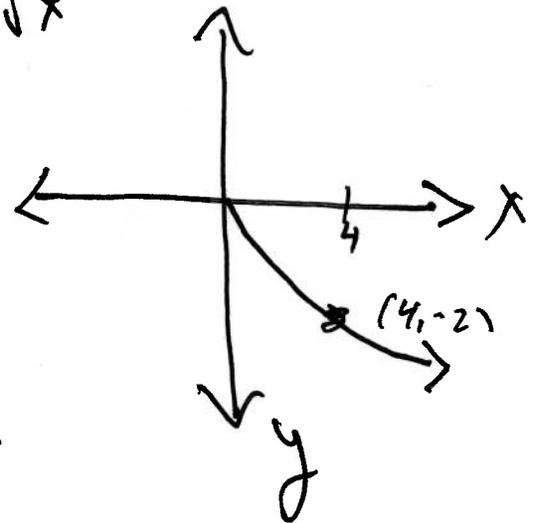
Reflections



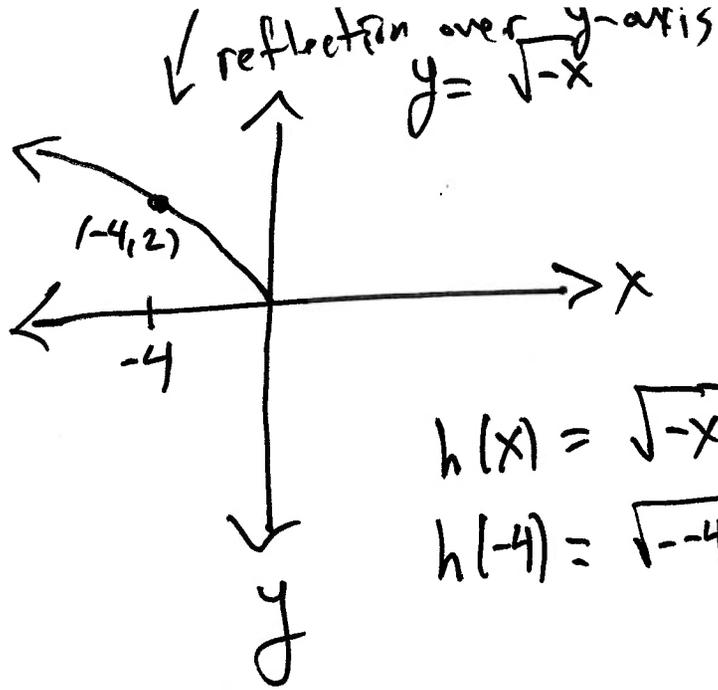
$$f(x) = \sqrt{x}$$
$$f(4) = \sqrt{4} = 2$$

reflection
over x-axis

$$y = -\sqrt{x}$$



$$g(x) = -\sqrt{x}$$
$$g(4) = -\sqrt{4} = -2$$



Rules for reflection

~~Graph of~~

$y = -f(x)$ reflects the graph of $f(x)$ over x -axis

$y = f(-x)$ reflects the graph of $f(x)$ over y -axis

Composition of Functions

Given two functions f, g , the composition of f and g is defined as

$$(f \circ g)(x) = f(g(x))$$

Example

$$f(x) = \sin(x), \quad g(x) = x^2 + 1$$

Determine $f \circ g$ and $g \circ f$.

$$\begin{aligned}
 f \circ g(x) &= f(g(x)) \\
 &= f(x^2 + 1) \\
 &= \sin(x^2 + 1)
 \end{aligned}$$

$$f \circ g(x) = \sin(x^2 + 1)$$

$$\begin{aligned}
 g \circ f(x) &= g(f(x)) \\
 &= g(\sin(x)) \\
 &= (\sin(x))^2 + 1 \\
 &= \sin^2(x) + 1
 \end{aligned}$$

$$g \circ f(x) = \sin^2(x) + 1$$

Note: $f \circ g(x) \neq g \circ f(x)$

$$\begin{aligned}
 g(x) &= x^2 + 1 \\
 f(x) &= \sin(x)
 \end{aligned}$$

~~two~~
 (*) Composition of ~~functions~~
 two types of functions
 from section 1.2
 can produce a
 new type of
 function. (*)

Decomposing Functions

Example Let $F(x) = (x^2+1)^3$. Find f and g such that
 $F(x) = f \circ g(x)$.

$$\begin{array}{c} \text{outside} \nearrow f(g(x)) = (x^2+1)^3 \\ \uparrow \\ \text{inside} \end{array}$$

"inside" x^2+1
"outside" $()^3$

$g(x) = x^2+1$
$f(x) = x^3$

check: $f(g(x)) = f(x^2+1) = (x^2+1)^3 \checkmark$

Example Let $F(x) = \sin(3x+2)$. Find f, g such as
that $F(x) = f \circ g(x)$.

$$f(g(x)) = \sin(3x+2)$$

"inside" $3x+2$
"outside" $\sin()$

$g(x) = 3x+2$
$f(x) = \sin(x)$

check: $f(g(x)) = \sin(3x+2) = \sin(3x+2) \checkmark$

Example $F(x) = \cos^2(x+9)$. Find f, g, h such that
 $F(x) = f \circ g \circ h(x)$.

$$f(g(h(x))) = (\cos(x+9))^2$$

aside: $\cos^2(x+9) = (\cos(x+9))^2$

$$(\cos(x+9))^2$$

most inside
second most inside
outside

$x+9$
 \cos
 $)^2$

$$\begin{aligned} h(x) &= x+9 \\ g(x) &= \cos(x) \\ f(x) &= x^2 \end{aligned}$$

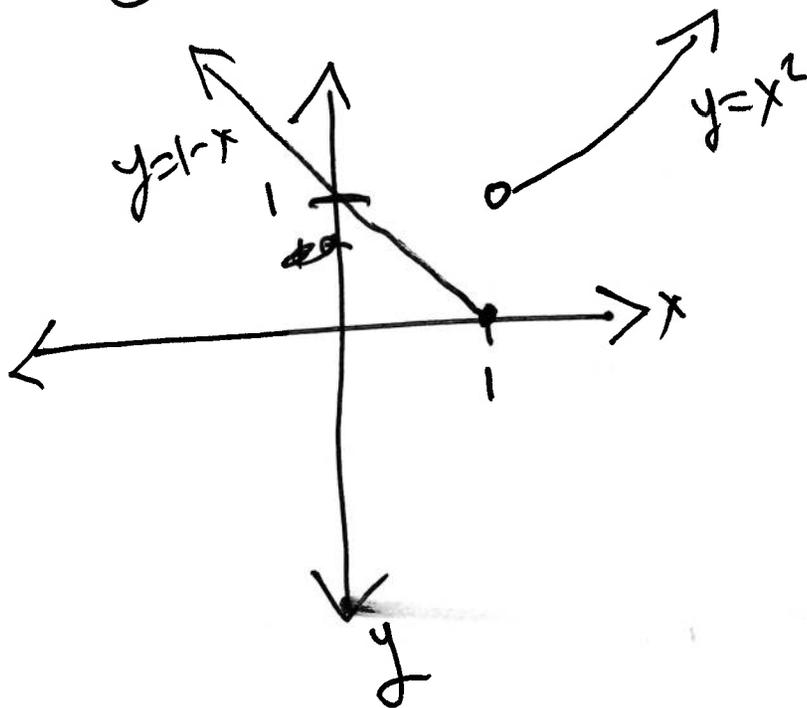
check: $f(g(h(x))) = f(g(x+9))$
 $= f(\cos(x+9))$
 $= (\cos(x+9))^2$
 $= \cos^2(x+9) \quad \checkmark$

Piecewise Defined Functions (from 1.1)

A piecewise defined function is a function defined by different formulas in different parts of the domain.

Example

$$f(x) = \begin{cases} 1-x & \text{if } x \leq 1 \\ x^2 & \text{if } x > 1 \end{cases}$$



some values

x	f(x)
1	$f(1) = 1 - 1 = 0$
2	$f(2) = 2^2 = 4$
0	$f(0) = 1 - 0 = 1$