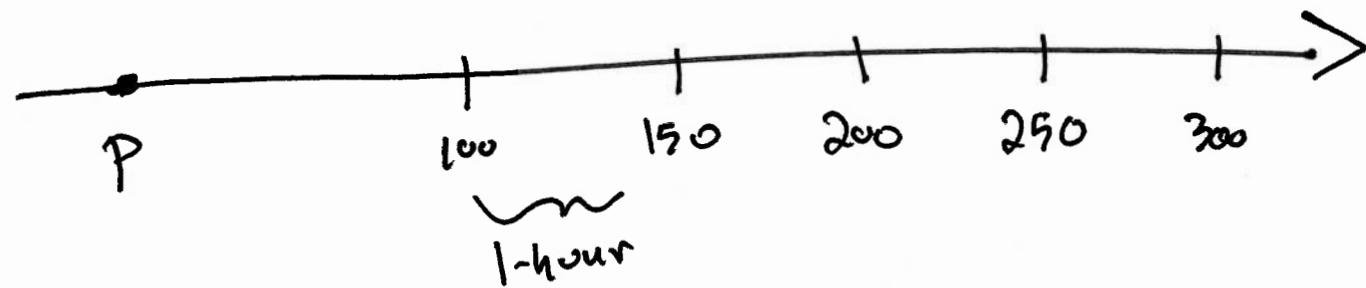


1.5 Exponential Functions

Linear function of time in "real-world":

Say you start 100 miles away from some point P and you travel 50 miles per hour in the direction away from. After t hours how far away from P are you?

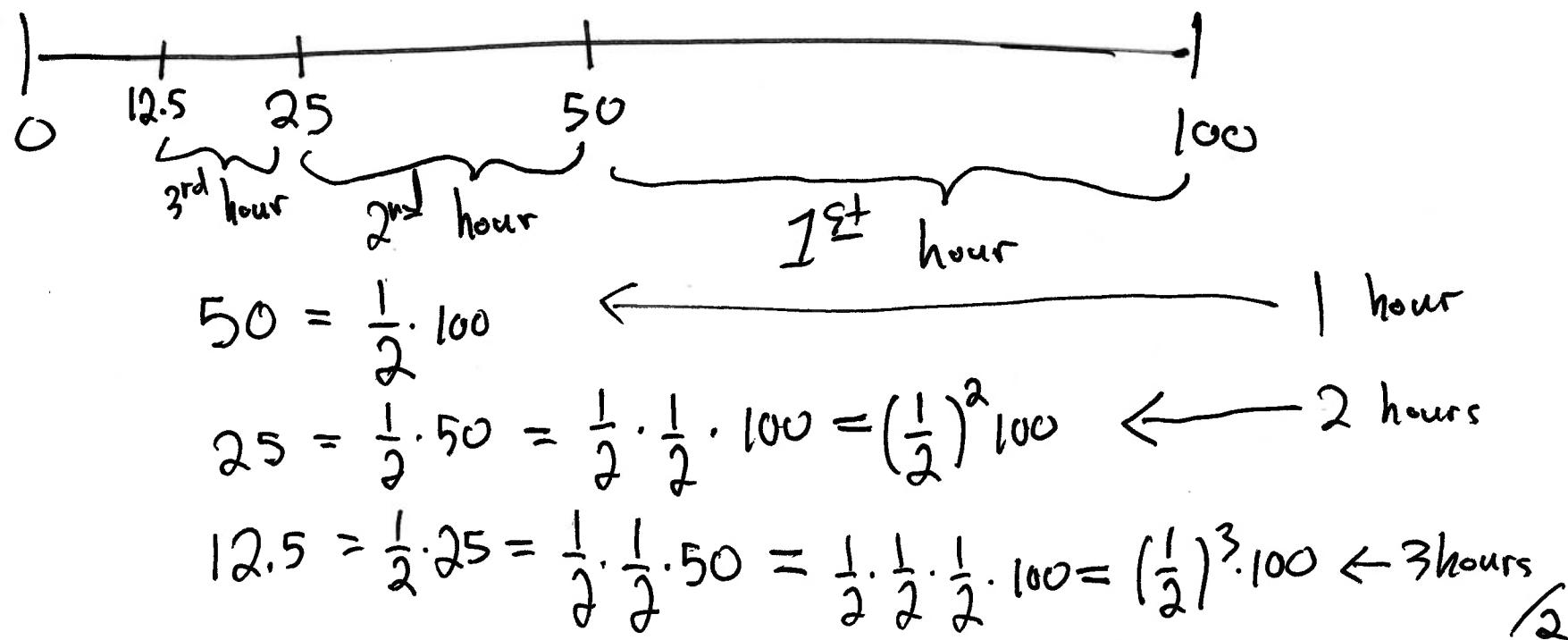


$f(t) = \cancel{100} 100 + 50t$ tells you how far away from P you are at time t.

For every hour that passes you add 50 miles.

Exponential function of time in "real-world":

Say you have 100 dollars in your bank account. Someone hacks your bank account and starts stealing your money. Every hour they steal half of the money that is in your bank account. After t -hours, how much money is in your bank account?



$f(t) = 100 \cdot \left(\frac{1}{2}\right)^t$ tells you how much money you have in your bank account after t hours

For every hour that passes you multiply the amount of money in your bank account by $\frac{1}{2}$.

$f(t) = 100 \cdot \left(\frac{1}{2}\right)^t$ is an example of an exponential function. It is called an exponential function because the variable, t , is in the exponent.

In general, an exponential function is a function of the form

$$f(x) = a^x$$

where a is a positive constant.

(More generally, $f(x) = a_0 \cdot a^x$ where a_0 is any real number. $f(0) = a_0 \cdot a^0 = a_0 \cdot 1 = a_0$, so a_0 is the initial value.)

Meaning of $f(x) = a^x$ for various x -values:
 $x=n$ where n is a positive integer

$$a^n = \underbrace{a \cdot a \cdots a \cdot a}_{n\text{-times}}$$

multiply a with itself
 n -times

$x=0$

$$a^0 = 1$$

$x = -n$ where n is a positive integer

$$a^{-n} = \frac{1}{a^n} = \underbrace{\frac{1}{a} \cdot \frac{1}{a} \cdots \frac{1}{a}}_{n\text{-times}}$$

$x = \frac{p}{q}$ where p and q are integers and $q > 0$

(so x is a fraction)

$$a^{\frac{p}{q}} = \boxed{\cancel{a^p}} \cdot \left(a^{\frac{1}{q}}\right)^p = (\sqrt[q]{a})^p$$

What is a^x if x is an irrational number?

(Irrational means x is not of form $x = \frac{p}{q}$.)

For example, what is $2^{\sqrt{3}}$ or 5^π ?



$$(2)^{3^{\frac{1}{2}}} = 2^{\sqrt{3}}$$

Say $f(x) = 2^x$ and we want to determine
 $f(\sqrt{3}) = 2^{\sqrt{3}}$.

$$\sqrt{3} = 1.73205080757\dots$$

$$1.7 = \frac{17}{10}$$

$$1.73 = \frac{173}{100}$$

$$1.732 = \frac{1732}{1000}$$

$$1.73205 = \frac{173205}{100000}$$

$$1.7320508 = \frac{17320508}{10000000}$$

These rational numbers (fractions) get closer and closer to $\sqrt{3}$.

Plays these rational numbers
into $f(x) = 2^x$

$$f(1.7) = 2^{17/10} = 3.249010\dots$$

$$f(1.73) = 2^{173/100} = 3.317278\dots$$

$$f(1.732) = 2^{1732/1000} = \dots$$

$$f(1.73205) = 2^{173205/100000} = 3.321995\dots$$

$$f(1.7320508) = 2^{17320508/100000000} = 3.321997\dots$$

$$f(1.732050807) = 2^{1732050807/10000000000} = 3.321997\dots$$

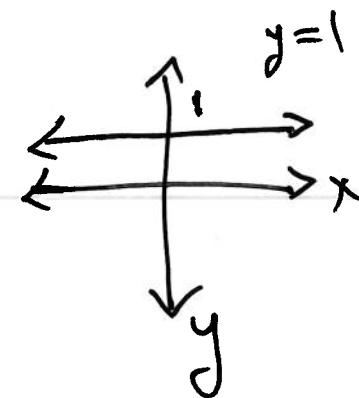
The values of f as the inputs get closer to $\sqrt{3}$ are
getting closer to some number $3.321997\dots$

If we do this forever, then we define the number we
are getting closer to to be $f(\sqrt{3}) = 2^{\sqrt{3}} = 3.32199708548\dots$

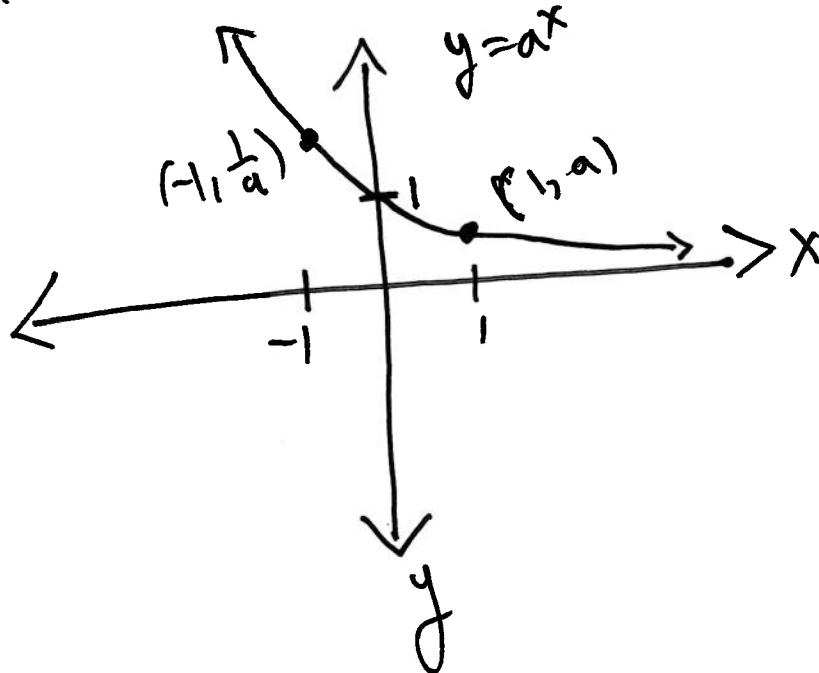
(we'll say more in section 2.2.)

Graphs of exponential functions

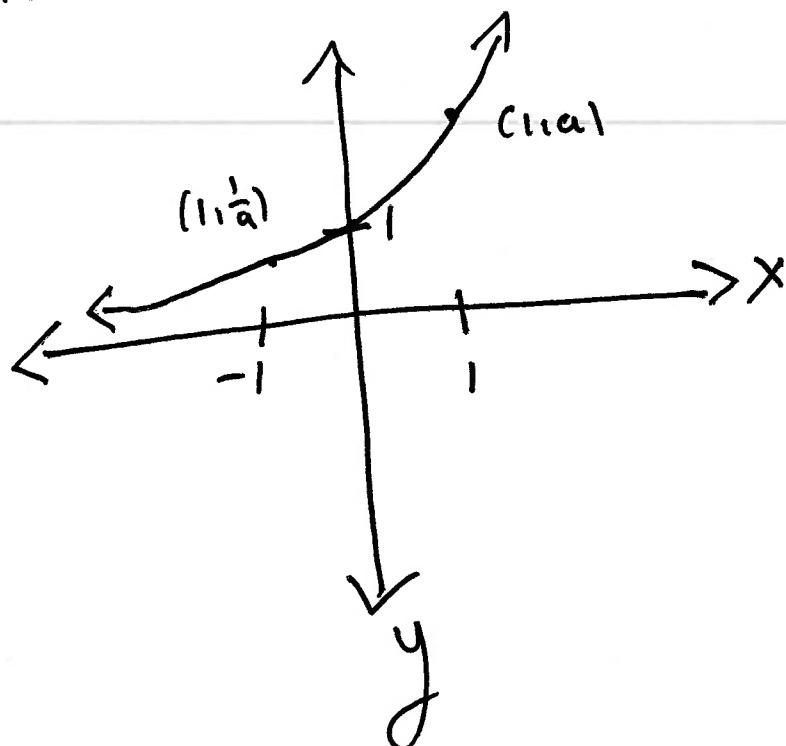
Type 1 $f(x) = a^x$ when $a = 1$
 $f(x) = 1^x = 1$



Type 2 $f(x) = a^x$ when $a < 1$



Type 3 $f(x) = a^x$ when $a > 1$



If $a > 1$ then $f(x) = a^x$, as x gets larger, grows faster than any polynomial function.

EX: $f(x) = 2^x$

$$f(2) = 2^2 = 4$$
$$f(5) = 2^5 = 32$$
$$f(10) = 2^{10} = 1024$$

~~Exponential~~ $g(x) = x^2$

$$g(2) = 2 \cdot 2 = 4$$
$$g(5) = 5 \cdot 5 = 25$$
$$g(10) = 10 \cdot 10 = 100$$

$$f(100) = 2^{100} = 1267650600228229401496703205376$$

(31 digits)

$$g(100) = 100 \cdot 100 = 10000$$

(4 digits)

Exponential Rules

a,b positive numbers

$$1. \quad a^{x+y} = a^x \cdot a^y$$

$$3. \quad (a^x)^y = a^{xy}$$

x,y real numbers

$$2. \quad a^{x-y} = \frac{a^x}{a^y}$$

$a^x \cdot a^{-y} = a^y$

$$4. \quad (ab)^x = a^x b^x$$