Reminder

Homework 3 is ~~to~~ turned in on gradescope.com

Midterm on Monday Jan 27

- covers 1.1, 1.2, 1.3, 1.5, 1.6, 2.1, 2.2
- 6 problems for 50 points
  - 3 problems from 1.1, 1.2, 1.3, 1.5, 1.6 worth 21 pts
  - 3 problems from 2.1, 2.2 worth 29 points
- To prepare
  - understand HW problems
  - do problems in book similar to HW problems
  - understand concepts and examples in book
  - " " from lecture

Example from 1.6

Determine the inverse of $f(x) = \dfrac{3x+1}{x-2}$.

$$y = \frac{3x+1}{x-2} \qquad (x-2)y = (x-2)\left(\frac{3x+1}{x-2}\right)$$

$$(x-2)y = 3x+1$$

$$xy - 2y = 3x+1$$

$$xy - 3x - 2y = 1$$

$$xy - 3x = 1+2y$$

$$x(y-3) = 1+2y$$

$$x = \frac{1+2y}{y-3}$$

switch x and y

$$y = \frac{1+2x}{x-3}$$

$$\boxed{f^{-1}(x) = \frac{1+2x}{x-3}}$$

$$\boxed{\cancel{f^{-1}(x) = \frac{1+2y}{y-3}}}$$

2.1 Velocity (speed)

$f(t)$ = function that tells you your distance away from a point at time t (position function)

in general, average velocity $= \dfrac{\text{distance traveled in time } t}{\text{time } t}$

with $f(t)$:

$$\text{average velocity from time } t_1 \text{ to time } t_2 = \frac{f(t_2) - f(t_1)}{t_2 - t_1}$$

Question: What is instantaneous velocity?

Example $f(t) = t^2$

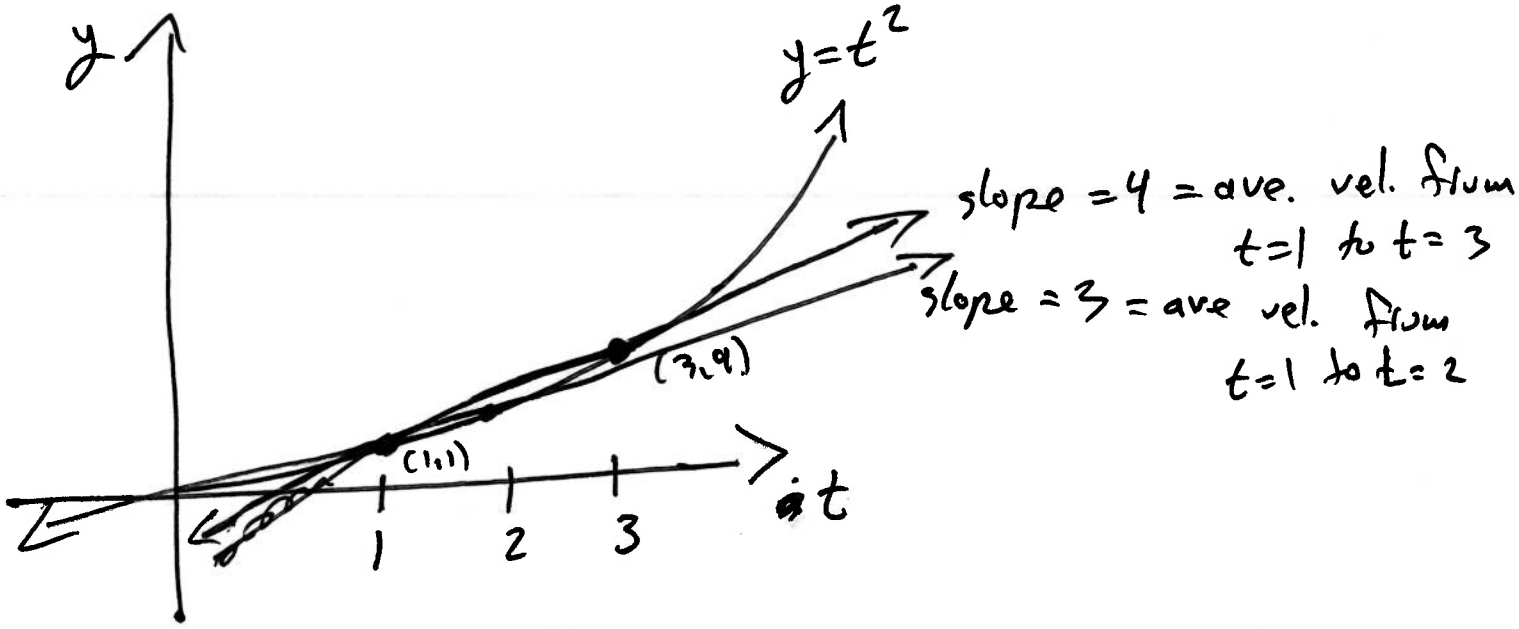
average velocity from time $t=1$ to time $t=3$:

$$\frac{f(3)-f(1)}{3-1} = \frac{3^2-1^2}{2} = \frac{8}{2} = 4$$

" " $t=1$

to time $t=2$:

$$\frac{f(2)-f(1)}{2-1} = \frac{2^2-1^2}{1} = 3$$



The average velocities from time $t=1$ to time $t=2$ or $t=3$ are estimates of the instantaneous velocity at time $t=1$.

To get a better estimate of instantaneous velocity at $t=1$, calculate the average velocity over a smaller time interval: $t=1$ to time ~~t=1+h~~ $t=1+h$ is a small time interval if h is small. Average velocity from time $t=1$ to time $t=1+h$:

$$\frac{f(1+h)-f(1)}{1+h-1} = \frac{(1+h)^2-1^2}{h}$$

$$= \frac{1+2h+h^2-1}{h}$$

$$= \frac{2h+h^2}{h}$$

$$= \frac{h(2+h)}{h}$$

$$= 2+h$$

$2+h$ = average velocity from time $t=1$ to time $t=1+h$

The smaller h is, the closer $2+h$ is to the instantaneous velocity at $t=1$.

In general, the average velocity from time t to time $t+h$ is given by $\dfrac{f(t+h)-f(t)}{h}$. The smaller h is, the better this ~~[illegible]~~ estimate is for the instantaneous velocity at time t.

2.2 The Limit of a Function

Definition We write

$$\lim_{x \to a} f(x) = L$$

and say "the limit of $f(x)$ as x approaches a equals L" if we can make the values of $f(x)$ arbitrarily close to L by taking x sufficiently close to a (on either sides of a) but not equal to a.

Limits from a table of values

EX: $f(x) = \dfrac{x-1}{x^2-1}$. Determine $\lim_{x \to 1} f(x)$ from the following table of values: (note we <u>cannot</u> plug in $x=1$, that is $x=1$ is <u>not</u> in the domain of f)

x	$f(x)$
0.5	0.666...
0.9	0.526...
0.99	0.502...
0.999	0.5002...
1	undefined
1.001	0.499...
1.01	0.497...
1.1	0.476...
1.5	0.400...

$\lim_{x \to 1} f(x) = 0.5$ because as x gets closer to 1, $f(x)$ gets closer to 0.5

When do limits not exist

It is possible that $\lim_{x \to a} f(x)$ does not exist.

Two possibilities:

1. Limit on left does not equal limit on right.
2. $f(x)$ oscillates as x approaches

Possibility 1

$$\lim_{x \to a^-} f(x) = L$$

(limit as x approaches a on the left equals L)

means we can make f(x) arbitrarily close to L by taking x sufficiently close to a and <u>less than a</u>.

$$\lim_{x \to a^+} f(x) = L$$

(right)

" "

and <u>greater than a</u>