Reminder

Homework 3 is due turned in on gradescope.com
Midterm on Monday Jan 27

- covers 1.1, 1.2, 1.3, 1.5, 1.6, 2.1, 2.2
- 6 problems for 50 points
  - 3 problems from 1.1, 1.2, 1.3, 1.5, 1.6 worth 21 pts
  - 3 problems from 2.1, 2.2 worth 29 points

- To prepare
  - understand HW problems
  - do problems in book similar to HW problems
  - understand concepts and examples in book
  - "from lecture"
Example from 1.6

Determine the inverse of \( f(x) = \frac{3x+1}{x-2} \).

\[
\begin{align*}
y &= \frac{3x+1}{x-2} \\
(x-2)y &= (x-2)\left(\frac{3x+1}{x-2}\right) \\
(x-2)y &= 3x+1 \\
x &= \frac{1+2y}{y-3} \\
\Rightarrow \quad y &= \frac{1+2x}{x-3} \\
f^{-1}(x) &= \frac{1+2x}{x-3}
\end{align*}
\]
2.1 Velocity (speed)

\[ f(t) = \text{function that tells you your distance away from a point at time } t \] (position function)

in general, average velocity = \( \frac{\text{distance traveled in time } t}{\text{time } t} \)

with \( f(t) \):

average velocity from time \( t_1 \) to time \( t_2 \) = \( \frac{f(t_2) - f(t_1)}{t_2 - t_1} \)

Question: What is instantaneous velocity?

Example \( f(t) = t^2 \)

average velocity from time \( t=1 \) to time \( t=3 \):

\[ \frac{f(3) - f(1)}{3 - 1} = \frac{3^2 - 1^2}{2} = \frac{8}{2} = 4 \]

" \( t=1 \)

average velocity from time \( t=2 \) to time \( t=3 \):

\[ \frac{f(3) - f(2)}{3 - 2} = \frac{3^2 - 2^2}{1} = 3 \]
The average velocities from time $t=1$ to time $t=2$ or $t=3$ are estimates of the instantaneous velocity at time $t=1$.

To get a better estimate of instantaneous velocity at $t=1$, calculate the average velocity over a smaller time interval: $t=1$ to time $(1+h)$ where $t=1+h$ is a small time interval if $h$ is small. Average velocity from time $t=1$ to time $t=1+h$:
\[ \frac{f(1+h) - f(1)}{1+h - 1} = \frac{(1+h)^2 - 1^2}{h} \]

\[ = \frac{1+2h+h^2 - 1}{h} \]

\[ = \frac{2h+h^2}{h} \]

\[ = h(2+h) \]

\[ = 2+h \]

\[ 2+h = \text{average velocity from time } t=1 \text{ to time } t=1+h \]

The smaller \( h \) is, the closer \( 2+h \) is to the instantaneous velocity at \( t=1 \).

In general, the average velocity is given by \( \frac{f(t+h) - f(t)}{h} \) from time \( t \) to time \( t+h \), The smaller \( h \) is, the better this numerical estimate is for the instantaneous velocity at time \( t \).
2.2 The Limit of a Function

Definition: We write
\[ \lim_{{x \to a}} f(x) = L \]
and say "the limit of \( f(x) \) as \( x \) approaches \( a \) equals \( L \)" if we can make the values of \( f(x) \) arbitrarily close to \( L \) by taking \( x \) sufficiently close to \( a \) (on either side of \( a \)) but not equal to \( a \).

Limits from a table of values

**EX:** \( f(x) = \frac{x-1}{x^2-1} \). Determine \( \lim_{{x \to 1}} f(x) \) from the following table of values: (note we cannot plug in \( x = 1 \), that is \( x = 1 \) is not in the domain of \( f \))
<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.666...</td>
</tr>
<tr>
<td>0.9</td>
<td>0.526...</td>
</tr>
<tr>
<td>0.99</td>
<td>0.502...</td>
</tr>
<tr>
<td>0.999</td>
<td>0.5002...</td>
</tr>
<tr>
<td>1</td>
<td>undefined</td>
</tr>
<tr>
<td>1.001</td>
<td>0.499...</td>
</tr>
<tr>
<td>1.01</td>
<td>0.497...</td>
</tr>
<tr>
<td>1.1</td>
<td>0.476...</td>
</tr>
<tr>
<td>1.5</td>
<td>0.400...</td>
</tr>
</tbody>
</table>

When do limits not exist

It is possible that $\lim_{x \to a} f(x)$ does not exist.

Two possibilities:

1. Limit on left does not equal limit on right.
2. $f(x)$ oscillates as $x$ approaches
Possibility 1

\[ \lim_{{x \to a^-}} f(x) = L \] (limit as \( x \) approaches \( a \) on the left)
equals \( L \)

\[ \lim_{{x \to a^+}} f(x) = L \] (right)

means we can make \( f(x) \) arbitrarily close to \( L \) by taking \( x \) sufficiently close to \( a \) and less than \( a \).

and greater than \( a \)