~~A few words about abelian groups~~ Direct Products of Groups

<u>Def</u>: Let ~~G, H be~~ $(G, *)$, $(H, \square)$ be two gps. We define the <u>cartesian</u> (or <u>direct</u>) <u>product</u> of G and H to be the group $(G \times H, \Delta)$, where

$$G \times H = \{ (g,h) : g \in G, h \in H \}$$ and Δ is defined by

$$(a,b) \,\Delta\, (c,d) = (a * b, b \,\square\, d)$$

<u>Exercise</u>: Show that this makes $G \times H$ into a gp.

<u>Examples</u>: Good ~~the~~ way to make abelian gps:

1. $\mathbb{R}^n = \underbrace{\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{n\text{-times}}$ is the Cartesian product of $\mathbb{R}$ with itself

2. $\mathbb{Z} \times \mathbb{Z} = \{ (a,b) : a,b \in \mathbb{Z} \}$, $\quad (a,b) + (c,d) = (a+c, b+d)$

3. $\mathbb{Z} \times \mathbb{Z}/5\mathbb{Z}$, $\quad (1,0)$ has infinite order
   $(0,\bar{1})$ has ~~[illegible]~~ order 5
   $(1,\bar{1})$ has infinite order

4. $\mathbb{Z}/4\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ $\quad (\bar{2},\bar{1})$ - has order 2

$$(\bar{2},\bar{1}) + (\bar{2},\bar{1}) = (\bar{4},\bar{2}) = (\bar{0},\bar{0})$$

~~5. $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/6\mathbb{Z}$ $(\bar{2},\bar{1})$ has order 6~~

5. $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \ni (\overline{2}, \overline{1})$ has order 6

$2\cdot(\overline{2},\overline{1}) = (\overline{2},\overline{1}) + (\overline{2},\overline{1}) = (\overline{1},\overline{0})$

$3\cdot(\overline{2},\overline{1}) = (\overline{6},\overline{3}) = (\overline{0},\overline{1})$

$4\cdot(\overline{2},\overline{1}) = (\overline{8},\overline{4}) = (\overline{2},\overline{0})$

$5\cdot(\overline{2},\overline{1}) = (\overline{10},\overline{5}) = (\overline{1},\overline{1})$

$6\cdot(\overline{2},\overline{1}) = (\overline{12},\overline{6}) = (\overline{0},\overline{0})$

Therefore $\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} = \langle(\overline{2},\overline{1})\rangle \cong \mathbb{Z}/6\mathbb{Z}$

$\mathbb{Z}/3\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$ is cyclic

6. $\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \ni (\overline{2}, \overline{2})$ has order 6

$2\cdot(\overline{2},\overline{2}) = (\overline{4},\overline{4}) = (\overline{4},\overline{0})$

$3\cdot(\overline{2},\overline{2}) = (\overline{6},\overline{6}) = (\overline{0},\overline{2})$

$4\cdot(\overline{2},\overline{2}) = (\overline{8},\overline{8}) = (\overline{2},\overline{0})$

$5\cdot(\overline{2},\overline{2}) = (\overline{10},\overline{10}) = (\overline{4},\overline{2})$

$6\cdot(\overline{2},\overline{2}) = (\overline{12},\overline{12}) = (\overline{0},\overline{0})$

$\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z} \supseteq \langle(\overline{2},\overline{2})\rangle \cong \mathbb{Z}/6\mathbb{Z}$

<u>Question:</u> Is $\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$ cyclic?

What are the subgps of $\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/4\mathbb{Z}$?