Math 103A W23 HW 4

11. If $a^{24} = e$, then order of a must be a factor of 24. So it can be 1,2,3,4,6,8,12,24.

26. **Suggested answer**:

26. We know that all the possible order of the subgroups must divide the order of the group. We also know that in case of Z_p, the order of the group is going to be p. From the given p is a prime. So the only divisors of p are 1 and itself, which makes the subgroups identity (trivial subgroup) and itself (the whole group).
$\therefore$ There are no other non-trivial. subgroup that are possible for Z_p if p is a prime number as there are no other divisors other than 1 and itself.

Comments:
The first sentence of the answer makes its idea clear - order of subgroup H must be a divisor of order of group $\mathbb{Z}/p$.

Common Mistakes:
Many proved the proposition by claiming H to be a cyclic group, which leads to a correct solution but is a little complicated.

27. **Suggested answer**:

27.)

$g, h \in G$ and $|g| = 15$, $|h| = 16$.

We know that for any subgroup/group, the order of every element divides the order of the subgroup/group.

This means that the orders of the elements of $\langle g \rangle$ can be 1, 3, 5, or 15, and $\langle h \rangle$ can be 1, 2, 4, 8, or 16.

Since the only possible order in both is 1, $\langle g \rangle \cap \langle h \rangle = \{e\}$.
The order is therefore 1.

Comments:
A straightforward and concise answer showing the only common divisor between the orders of g and h.

Common Mistakes:
Many proved the proposition by claiming $\langle g \rangle \cap \langle h \rangle$ to be a subgroup of both $\langle g \rangle$ and $\langle h \rangle$. This is correct but requires extra proof (no point is deducted because of this).

31. **Suggested answer**:

Proof. Suppose G is an abelian group.
Let H represent the elements of finite order in G.
We will show H is a subgroup.

Closure:
Suppose $a, b \in H$.
By definition $a, b \in G$ and thus $ab \in G$.
Furthermore, as a, b have finite order, we have $a^{k_1} = e$ and $b^{k_2} = e$ for some positive integers k_1, k_2.
It follows that $a^{k_1 k_2} = e$ and $b^{k_1 k_2} = e$.
Therefore, we have $a^{k_1 k_2} b^{k_1 k_2} = e$.
As G is abelian, we may reorder terms such that $(ab)^{k_1 k_2} = e$.
Therefore, we have shown ab has finite order and thus $ab \in H$.

Identity:
We know that $e^1 = e$ by definition and thus e has finite order.
Thus $e \in H$.

Inverse:
Suppose $a \in H$.
Then, as $a \in G$ by definition, we have that $a^{-1} \in G$.
By $Q23A$, the order of a is the same as the order of a^{-1} and thus a^{-1} has finite order.
Therefore, by definition $a^{-1} \in H$.

Thus H is a subgroup of G.

$\square$

Comments:
The elements of the proof are very clear: identity, inverse and closure.