Homework #1

1. - First check whether the function is continuous in the interval and has opposite signs at the endpoints. If so, the bisection method will work. Otherwise, go to next step.
   - If the function does not have opposite signs at the endpoints, then bisection method will not work, though you can do a further check of the midpoint to see if by luck you get a subinterval that has opposite signs at the endpoints, which would allow bisection method to continue. Otherwise go to next step.
   - If the function has opposite signs at the endpoints, then you can perform bisection method and see what the sequence converges to. Note, it may converge to a point with a discontinuity instead of a root.

2. (a) $c_0 = 5/2$, $c_1 = 11/4$, $c_2 = 23/8$, $c_3 = 45/16$.

   (b) $|r - c_3| \leq 1/16$.

3. The next bracketing interval is $[-3, -1/2]$, and only one root, $r = -2$ exists in that interval, so bisection method, since f is continuous everywhere and has opposite signs at -3 and $-1/2$, will converge to that root.

4. (a)
$$|r - c_{30}| \leq \frac{5}{2^{31}}.$$

   (b) Solve
$$\frac{5}{2^{n+1}} < 10^{-11}$$
   for n by taking log on both sides to get that $n \geq 38$.

5. (a)
   - First show
$$|c_0 - c_1| = (b_0 - a_0)/4.$$
     This solves for $(b_0 - a_0)/2$.
   - Then show $a_0 = c_0 - (b_0 - a_0)/2$ and $b_0 = c_0 - (b_0 - a_0)/2$.

   (b) $a_0 = -1.2$ and $b_0 = 0.8$.

6. (a)
   - First show bisection method with starting interval $[\alpha, \beta]$ means all approximations will be in (α, β).
   - Then use this to show, by contradiction, that the first k intervals generated by bisection method on f_1 starting with $[a, b]$ are the same as those generated by bisection method on f_2 starting with $[a, b]$, since otherwise $c_k \neq d_k$.
   - Then show this implies $c_i = d_i$, for all $i < k$, since midpoints of the same intervals.

(b) Let $f_1(x) = x + 0.7$, $f_2(x) = x - 0.7$, $a = -1$, $b = 1$, $k = 0$, and note $c_1 = -0.5$ and $d_1 = 0.5$.

7. (a)
   - Note cut locations in the interval split the interval so that 3/4 of it is on the left of the cut, and 1/4 is on the right. So worst case for interval length $[a_n, b_n]$ is $3(b_n - a_n)/4$, and iterating, worst case is $(3/4)^n(b-a)$.
   - Then, worst case for absolute error in an interval this size is $(1/4)(3/4)^n(b-a)$, since approximation splits the interval so that 1/3 of it is on the left, and 2/3 is on the right.

   (b) $c_0 = \pi/6$, though $cut_0 = 3\pi/8$, and so $c_1 = \pi/8$.

8. (a) True

   (b) False

   (c) False

   (d) True

   (e) False (assuming g has opposite signs at a, b)

9. (Matlab) See Matlab solutions.