Homework #4

1. Note $f \in C[a,b]$, look at the signs of $f(a)$ and $f(b)$, apply the Intermediate Value Theorem to f to show it has a root in $[a,b]$, and show this root is a fixed point of F.

2. (a) Use the definition of contractive mapping on $|F(s) - F(t)|$, and also use the definition of fixed points $F(s) = s, F(t) = t$.

   (b) Use the definition of the Mean Value Theorem on $|F(s) - F(t)|$, and also use the definition of fixed points $F(s) = s, F(t) = t$.

3. (a) Plug in to show $f(1) = 0$, and plug in to show $F_i(1) = 1$, for each $i = 1, 2, 3$.

   (b) Compute $|F_i'(1)|$. If it is > 1, then the fixed point iterations will in general not convergence, even when initial guesses are close to 1; if it is < 1, then fixed point iterations will converge if initial guesses are close enough to 1. For order of convergence, find the largest k such that $F^{(j)}(1) = 0$ for $j < k$ and $F^{(k)}(1) \neq 0$.

4. (a) Plug r into $F(x)$ and show $F(r) = r$.

   (b) Plug r into $F'(x)$ and show $F'(r) = 0$. This means the order of convergence is > 1.

5. (a) Plug $1, 2, 3$ into p to show $p(1) = p(2) = p(3) = 0$.

   (b) $q(x) = 2x^2 - 7.8x + 5.62$, with $R = -0.198$.

   (c) Quadratic formula says $q(x)$ has roots at 2.9462 and 0.95376.

6. (a)
$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

   (b)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

   (c)
$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 3 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}$$

7. (Matlab) See Matlab solutions.