Homework #5

1. (a) Use Lagrange form:

$$p_2(x) = -\frac{x(x+1)}{2} + \frac{(x+2)(x+1)}{2} - 3(x+2)x$$

   (b) Follows Newton form: use $a = -2, b = 0, c = -1$, so the original three data points are still being interpolated. Then solve for K using $p_3(1) = -1$ from the new data point to get $K = 1$.

   (c) Use your form for $p_3(x)$, also take its derivative, and plug in to get $p_3(-0.7)$ and $p_3'(-0.7)$.

2. (a) First get the interpolation polynomial for the $n+1$ data points, call it $p(x)$. Newton's form for adding a data point creates the polynomial

$$p(x) + K(x - x_0) \cdot \ldots \cdot (x - x_n),$$

   where $x_0, \ldots, x_n$ are the nodes of the $n+1$ data points. Then verify that for $K \neq 0$, these are all degree $n+1$ polynomials interpolate the $n+1$ data points.

   (b) Use Newton's form and add more data points to see there are an infinite number of polynomials of degree p that interpolate the $n+1$ data points.

   (c) Use your results to actually write down two polynomials.

3. (a)

$$\begin{aligned} p_2(x) &= f(x_0 - h)\frac{(x-x_0)(x-x_0-h)}{(-h)(-2h)} + f(x_0)\frac{(x-x_0+h)(x-x_0-h)}{h(-h)} + \\ &\quad f(x_0+h)\frac{(x-x_0+h)(x-x_0)}{2h(h)}, \end{aligned}$$

   (b) Take a derivative of $p_2(x)$ and plug in x_0 to get $A = 1/(2h)$, $B = 0$, and $C = -1/(2h)$.

   (c) Take two derivatives of $p_2(x)$ and plug in x_0 to get $A = 1/h^2$, $B = -2/h^2$, and $C = 1/h^2$.

4. (a) Get

$$p(x) = e^{0.1}\frac{(x-0.5)(x-0.6)}{(-0.4)(-0.5)} + e^{0.5}\frac{(x-0.1)(x-0.6)}{(0.4)(-0.1)} + e^{0.6}\frac{(x-0.1)(x-0.5)}{(0.5)(0.1)}$$

   and simplify.

   (b) Look at critical points of $f'''(x)$ (there are none) and endpoints and see which has the largest $|f'''(x)|$ value. The answer for the value is $e^{0.6}$.

(c)
$$|f(0.2) - p(0.2)| \le \frac{e^{0.6}}{3!}(0.1)(0.3)(0.4).$$

(d) Look at critical points of $(x-0.1)(x-0.5)(x-0.6)$ and endpoints and see which has the largest $|(x-0.1)(x-0.5)(x-0.6)|$ value. The answer for the value is approximately 0.0131.

(e)
$$|f(x) - p(x)| \le \frac{e^{0.6}}{3!}0.0131.$$
for all $x \in [0.1, 0.6]$.

5. (a) We have existence of a polynomial of degree $\le n$ that interpolates the data points, so the polynomial of least degree has to be less than or equal to this, giving its degree as $\le n$.

   (b) We have uniqueness of a polynomial of degree $\le n$ that interpolates the data points, so the polynomial of least degree has to be exactly that polynomial.

6. In order to show a polynomial is the interpolation polynomial for a set of data points, just make sure it has the correct degree and that it interpolates the data points.

7. (Matlab) See Matlab solutions.