Math 170B Midterm 1

May 3, 2017

- Please put your name, ID number, and sign and date.
- There are 4 problems worth a total of 100 points.
- **You must show your work to receive credit**.

Print Name: _____

Student ID: _____

Signature and Date: _____

Problem	Score
1	/25
2	/25
3	/25
4	/25
Total	/100

1. (25 pts) Let $f(x)$ be a continuous function and suppose we are **already given** the Matlab function "f.m", with header "function [y] = f(x)", that returns values of $f(x)$. Given the following header for a Matlab function:

   function [pN] = falseposition(c,d,N)

   complete the function so that it outputs the approximation p_N, of the method of **false position**, using initial guesses $p_0 = c, p_1 = d$. You may assume $c < d$ and $f(x)$ has different signs at c and d, however, make sure your program uses **at most** N function evaluations.

2. (25 pts) Circle **True** or **False** for the following questions. You do **not** need to show your work for this problem.

   (a) Let $f(x)$ be continuous and $a < b$ such that f has different signs at a and b. Then the sequence of approximations generated by **bisection** method on $f(x)$ using starting interval $[a, b]$ is always **the same** as the sequence of approximations generated by **bisection** method on $Cf(x)$ using starting interval $[a, b]$, for any $C < 0$.

   True or **False**

   (b) Let $f(x)$ be a twice continuously differentiable function and $a < b$ such that f has different signs at a and b. Then the sequence of approximations generated by the method of **false position** using $p_0 = a, p_1 = b$ is always **the same** as the sequence of approximations generated by **secant** method using $p_0 = a, p_1 = b$.

   True or **False**

   (c) The sequence of approximations generated by **Newton's** method on $f(x) = x^k$, for any integer $k \geq 1$, will always **converge quadratically** to the root at $x = 0$ when $p_0 \neq 0$ is close enough to 0.

   True or **False**

   (d) The sequence $4 + 2^{-3^n}$ **converges** and has **order** of convergence 3.

   True or **False**

   (e) **Fixed point** iterations always **converge** to a fixed point for any inital guess p_0 when the fixed point function is of the form $g(x) = Ax + B$.

   True or **False**

   (f) Let $f(x) = Ax + B$, with $A \neq 0$, and let $a < b$ be such that f has different signs at a and b. Then the approximation p_2 of the method of **false position** on $f(x)$, using $p_0 = a, p_1 = b$, must **satisfy** $f(p_2) = 0$.

   True or **False**

   (g) Let $f(x) = Cx^k$ for $C \neq 0$ and integer $k \geq 1$. The approximation p_1 of **modified Newton's** method, using any $p_0 \neq 0$, must **satisfy** $f(p_1) = 0$.

   True or **False**

   (h) Let $g(x)$ be a continuous function and let $a < b$ with $g(a) < a$ and $g(b) > b$. Then $g(x)$ must have **at least one** fixed point in the interval $[a, b]$.

   True or **False**

3. (25 pts) Consider the equation $y^3 + 2y^2 - y = x$ that **implicitly** defines y as a function of x ($y = y(x)$) for $x \in [4, 6]$. Approximate $y(6)$ with y_3 generated by **secant** method using initial guesses $y_0 = 0, y_1 = 1$.

4. (25 pts) Let $f(x)$ be an infinitely continuously differentiable function and suppose it has a root at p of **multiplicity** 3. Prove if the iterations given by

$$p_{n+1} = p_n - s\frac{f(p_n)}{f'(p_n)}$$

converge to p, when given initial guess p_0, then it converges **linearly**. In addition, find the **asymptotic error constant**.