Global Newlander-Nirenberg theorem for domains with $C^2$ boundary

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We consider two natural generalizations of the Newlander-Nirenberg theorem under the presence of a $C^2$ strictly pseudoconvex boundary. When a given formally integrable complex structure $X$ is defined on the closure of a bounded strictly pseudoconvex domain $D$ in $\mathbb{C}^n$ with $C^2$ boundary, we show the existence of global holomorphic coordinate systems defined on $\overline{D}$ that transform $X$ into the standard complex structure provided that $X$ is sufficiently close to the standard complex structure. Moreover, we show that such closeness is stable under a small $C^2$ perturbation of the boundary of $D$. As a consequence, when a given formally integrable complex structure is defined on a one-sided neighborhood of some point in a $C^2$ real hypersurface $M$ in $\mathbb{C}^n$, we prove the existence of local one-sided holomorphic coordinate systems provided that $M$ is strictly pseudoconvex with respect to the given complex structure. We also obtain results when the structures are finite smooth. This is joint work with Chun Gan.