MATH 109 Winter 2020 — Midterm I Study Guide

The following provides a list of concepts that you should be familiar with for the first midterm. It consists of essential points we have covered as well as some examples that appeared in lecture or in the homework together with occasional comments. You should also refer to the textbook, lecture notes and homework for an idea of what you may be expected to know. Reviewing examples done in lecture and assigned as homework will provide you with a solid understanding of concepts which may be tested. However, you may be asked to apply understanding of these concepts in new ways on the exam, so it is important that you master the underlying concepts and fully understand the motivation of each step of the solution in addition to knowing how to solve the exercises you review.

Chapter 1 - The language of mathematics

• Propositions, predicates, statements.
• The connectives ‘or’, ‘and’ and ‘not’.
• Truth tables.

Chapter 2 - Implications

• Truth table of $P \Rightarrow Q$.
• Language associated to implications: converse, necessary and sufficient conditions, if and only if.
• Equivalence of two statements.
• Divisibility.
• Even and odd integers.

Chapter 3 - Proofs

• Direct proofs.
• Proofs by cases.
• Constructing proofs backwards.
• Examples with algebra of real numbers.

Chapter 4 - Proofs by contradiction

• Proofs by contradiction.
• Proofs by contrapositive.
• Examples with arithmetic and algebra.
Chapter 5 - Induction

• The principle of induction.
• Examples with inequalities and divisibility.
• Definitions by induction.
• Examples with sum notation and factorial.

Chapter 6 - Sets

• Sets, elements of sets, subsets, equality of sets, empty set.
• Union, intersection, difference of two sets.
• The power set.
• Properties of union, intersection and complement.
• Using truth tables to prove statements about sets.
• Venn diagrams.
• Constructing proofs for statements about sets.

Chapter 7 - Quantifiers

• Universal statements (\(\forall\)) and existential statements (\(\exists\)).
• Proving and disproving universal and existential statements and combinations thereof.
• The Cartesian product of two sets.