Math 109 | **Midterm Exam 1** | **Name:** ______________
April 19, 2019 | **(Total points: 20)** | **PID:** ______________

Instructions
1. Write your Name and PID in the spaces provided above.
2. Make sure your Name is on every page.
3. Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.
4. Note that since this class is about proofs, every statement should be proved. The only exceptions are statements that were proven in the text-book or in class.

1. (5 points) Check all the statements that are equal to $p \implies (q \vee r)$ (in this question only the answers will be graded.)

   ☐ $(p \vee q) \implies \neg q \wedge r$

   ☐ $(p \wedge q \wedge r) \iff q \wedge r$

   ☐ $\neg p \wedge (q \vee r)$

2. (5 points) Consider a set G equipped with an operation $\cdot$ on the elements of G which satisfy the following axioms:

   1. For any $a, b, c \in G$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
   2. There is a unique $e \in G$ such that for any $a \in G$, $e \cdot a = a = a \cdot e$ (such an element is called the identity element).
   3. For every $a \in G$ there exists $b \in G$ such that $a \cdot b = e$, where e is the identity element.
   4. For every $a \in G$ there exists $b \in G$ such that $b \cdot a = e$, where e is the identity element.

   Let e be the identity element. Show that if $a \cdot b = e$, then $b \cdot a = e$.

3. (5 points) For an integer n define $n!$ as follows: $1! = 1$ and $n! = (n-1)! \cdot n$. Show that $n! \geq 2^n$ for any $n \geq 4$.

4. (5 points) Show that there does not exist a largest integer.