Math 109 **Midterm Exam 2** **Name:** _____
May 17, 2019 **(Total points: 20)** **PID:** _____

Instructions
1. Write your Name and PID in the spaces provided above.
2. Make sure your Name is on every page.
3. Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.
4. Note that since this class is about proofs, every statement should be proved. The only exceptions are statements that were proven in the text-book or in class.

1. (5 points) Check all the correct statements (in this question only the answers will be graded).

   ☐ $\exists m \in \mathbb{N} \forall n \in \mathbb{N} m \geq n$.

   ☐ For any two sets A, B, then $A \subseteq B$ if and only if $A \cup B = B$.

   ☐ The cardinality of the set $([n])_{\{0,1\}}$ is $n(n-1)$

2. (5 points) Let $n \in \mathbb{N}$ be odd. Show that

$$\left|\left\{S \subseteq [n] \;\middle|\; |S| < \frac{n}{2}\right\}\right| = \left|\left\{S \subseteq [n] \;\middle|\; |S| > \frac{n}{2}\right\}\right|$$

by constructing a bijection.

3. (5 points) How many numbers from [999] are divisible by 3 or by 5?

4. (5 points) Let n be a positive integer.

   (a) Let a and b be integers. Show that if a and b have the same reminder when divided by n, then $a - b$ is divisible by n.

   (b) Prove that for every integers $a_0, \ldots, a_n$ there are $0 \leq i < j \leq n$ such that $a_i - a_j$ is divisible by n.