Name: $\qquad$ PID: $\qquad$

- Print your NAME and your PID on the first page.
- No credit will be given for unsupported answers, even if correct.
- This is an open book exam which means that you can use the textbook, lecture notes and your homework as assistance. But no any electronic devices are allowed and you cannot use the internet, except for the purpose of writing solutions.
- This exam has 4 questions with a maximum of 22 points.

1. ( 5 pt ) Find the explicit solution of the initial value problem

$$
x y^{\prime}+2 y=\sin x, \quad y\left(\frac{\pi}{2}\right)=1
$$

2. ( 5 pt ) Solve the following problem. Leave your answer in implicit form.

$$
\left(e^{y}+1\right) d x+\left(2 y+x e^{y}\right) d y=0
$$

3. ( 5 pt ) Solve the initial value problem.

$$
9 y^{\prime \prime}-6 y^{\prime}+y=0, \quad y(0)=2, y^{\prime}(0)=1
$$

4. ( 7 pt ) All that is known concerning a second-order constant-coefficient differential equation $y^{\prime \prime}+p y^{\prime}+q y=f(t)$ is that $t^{2}+1+e^{t} \cos t, t^{2}+1+e^{t} \sin t$, and $t^{2}+1+e^{t} \cos t+e^{t} \sin t$ are solutions.
(a) Determine two linearly independent solutions to the corresponding homogeneous equation (you don't need to prove linear independence).
(b) Find a suitable choice of $p, q$ and $f(t)$ that enables these solutions.
