Math 18 | **Midterm Exam 2 v.1** | **Name:** _____
October 19, 2018 | **(Total points: 21)** | **PID:** _____

Instructions
1. Write your Name and PID in the spaces provided above.
2. Make sure your Name is on both sides of every page.
3. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
4. You may use one handwritten page of notes, but no books or other assistance during this exam.
5. Read each question carefully and answer each question completely.
6. Write your solutions clearly in the spaces provided; only work on this exam booklet will be graded.
7. Show all of your work. No credit will be given for unsupported answers, even if correct.

1. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

2. (6 points) Let $U, V \subseteq \mathbb{R}^4$ be vector spaces.

   For each statement write **T** if it is always True; write **F** if it is ever False. 2 points will be assigned for each correct response, 1 point for each blank non-response, and 0 points for each incorrect response. No justification is required.

   (a) ___ If U is a subspace of V and $\dim U = \dim V$, then $U = V$.

   (b) ___ If U is spanned by 2 vectors and V is spanned by 3 vectors, then $\dim U \leq \dim V$.

   (c) ___ The set $S = \{u \mid u \in U \text{ and } u \in V\}$ is a subspace of U.

3. Let

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}.$$

such that $\det A = 5$.

(a) (3 points) Let

$$B = \begin{pmatrix} a+d & d & g \\ 2b+2e & 2e & 2h \\ c+f & f & i \end{pmatrix}.$$

What is the determinant of B? Justify your answer.

(b) (3 points) Let

$$C = \begin{pmatrix} a & b & c & 1 \\ d & e & f & 2 \\ g & h & i & 1 \end{pmatrix}.$$

What is $\operatorname{rank} C$? Justify your answer.

4. Let A be a 3×3 matrix.

   (a) (2 points) Assume that the rows of A are linearly independent. Find a basis for $\mathrm{Col}\,(A)$. Justify your answer.

   (b) (2 points) Assume that the columns of A span $\mathbb{R}^3$. Find a basis for $\mathrm{Null}\,(A)$. Justify your answer.

5. Let A, B be 6×6 matrices.

   (a) (2 points) Assume A satisfies $5A^3 - 4A^2 + 3A - 6I = 0$. Find A^{-1}.

   (b) (2 points) Assume B satisfies $B^3 = 0$. Find all the eigenvalues of B. Justify your answer.