1. (1 point) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

2. (6 points) Let $U, V \subseteq \mathbb{R}^4$ be vector spaces.

   For each statement write T if it is always True; write F if it is ever False. 2 points will be assigned for each correct response, 1 point for each blank non-response, and 0 points for each incorrect response. No justification is required.

   (a) ___ If $U$ is a subspace of $V$ and $\dim U = \dim V$, then $U = V$.

   (b) ___ If $U$ is spanned by 2 vectors and $V$ is spanned by 3 vectors, then $\dim U \leq \dim V$.

   (c) ___ The set $S = \{u \mid u \in U \text{ and } u \in V\}$ is a subspace of $U$. 

3. Let
\[
A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}.
\]
such that \( \det A = 5 \).

(a) (3 points) Let
\[
B = \begin{pmatrix} a + d & d & g \\ 2b + 2e & 2e & 2h \\ c + f & f & i \end{pmatrix}.
\]
What is the determinant of \( B \)? Justify your answer.

(b) (3 points) Let
\[
C = \begin{pmatrix} a & b & c & 1 \\ d & e & f & 2 \\ g & h & i & 1 \end{pmatrix}.
\]
What is rank \( C \)? Justify your answer.
4. Let $A$ be a $3 \times 3$ matrix.

   (a) (2 points) Assume that the rows of $A$ are linearly independent. Find a basis for $\text{Col}(A)$. Justify your answer.

   (b) (2 points) Assume that the columns of $A$ span $\mathbb{R}^3$. Find a basis for $\text{Null}(A)$. Justify your answer.
5. Let $A, B$ be $6 \times 6$ matrices.

(a) (2 points) Assume $A$ satisfies $5A^3 - 4A^2 + 3A - 6I = 0$. Find $A^{-1}$.

(b) (2 points) Assume $B$ satisfies $B^3 = 0$. Find all the eigenvalues of $B$. Justify your answer.