Every Counterexample In Topology Appearing In The Book "Counterexamples In Topology" by Lynn Steen and J. Arthur Seebach, Jr.

And Whether Or Not Each One Is Compact

Tanny Libman[1]

[1]The "Quiet" Office On The 6th Floor Of APM, UCSD – APM6446
nlibman@ucsd.edu

October 26, 2021

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Section 1

Definition

Definition

A topological space X is compact if every open cover of X has a finite subcover

Section 2

Every Counterexample

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1. Finite Discrete Topology

Definition:

- Every subset is open

1. Finite Discrete Topology

Is it compact?

- Yes

2. Countable Discrete Topology

Definition:

- Every subset is open

2. Countable Discrete Topology

Is it compact?

- No

3. Uncountable Discrete Topology

Definition:

- Every subset is open

3. Uncountable Discrete Topology

Is it compact?

- No

4. Indiscrete Topology

Definition:

- Only open sets are X and $\varnothing$

4. Indiscrete Topology

Is it compact?

- Yes

5. Partition Topology

Definition:

- Any partition of a set X (along with $\varnothing$) defines a basis of a topology, called the partition topology

5. Partition Topology

Is it compact?

- Depends on the set and the partition

6. Odd-Even Topology

Definition:

- Partition topology on $\mathbb{Z}$ where the elements of the partition are $\{2k-1, 2k\}$

6. Odd-Even Topology

Is it compact?

- No

7. Deleted Integer Topology

Definition:

- X is the union of the open intervals $(n-1, n)$, and the topology on X is generated by the partition $\{(n-1, n)\}$

7. Deleted Integer Topology

Is it compact?

- No

8. Finite Particular Point Topology

Definition:

- Open sets are $\varnothing$ and any subset of X which contains a particular point p.

8. Finite Particular Point Topology

Is it compact?

- Yes

9. Countable Particular Point Topology

Definition:

- Open sets are $\varnothing$ and any subset of X which contains a particular point p.

9. Countable Particular Point Topology

Is it compact?

- No

10. Uncountable Particular Point Topology

Definition:

- Open sets are $\varnothing$ and any subset of X which contains a particular point p.

10. Uncountable Particular Point Topology

Is it compact?

- No

11. Sierpinski Space

Definition:

- $X = \{0, 1\}$ with open sets $\{\varnothing, \{0\}, X\}$.

11. Sierpinski Space

Is it compact?

- Yes

12. Closed Extension Topology

Definition:

- Let (X, τ) be any nonempty space, and let p be a point not in X. We define $X^* = X \cup \{p\}$, and the topology on X^* has that a set is open iff it is the empty set or is of the form $U \cup \{p\}$ for $U \in \tau$

12. Closed Extension Topology

Is it compact?

- N/A

13. Finite Excluded Point Topology

Definition:

- X is open, as is any subset of X which does not contain a given point $p \in X$

13. Finite Excluded Point Topology

Is it compact?

- Yes

14. Countable Excluded Point Topology

Definition:

- X is open, as is any subset of X which does not contain a given point $p \in X$

14. Countable Excluded Point Topology

Is it compact?

- Yes

15. Uncountable Excluded Point Topology

Definition:

- X is open, as is any subset of X which does not contain a given point $p \in X$

15. Uncountable Excluded Point Topology

Is it compact?

- Yes

16. Open Extension Topology

Definition:

- Let (X, τ) be a nonempty topological space, and let p be a point not in X. We define $X^* = X \cup \{p\}$, and say that a subset of X^* is open iff it is X^* or in τ

16. Open Extension Topology

Is it compact?

- N/A

17. Either-Or Topology

Definition:

- $X = [-1, 1]$ and a subset of X is open iff it either does not contain $\{0\}$ or does contain $(-1, 1)$

17. Either-Or Topology

Is it compact?

- Yes

18. Finite Complement Topology on a Countable Space

Definition:

- Open sets are those with finite complements, together with $\varnothing$ (and X)

18. Finite Complement Topology on a Countable Space

Is it compact?

- Yes

19. Finite Complement Topology on an Uncountable Space

Definition:

- Open sets are those with finite complements, together with $\varnothing$ (and X)

19. Finite Complement Topology on an Uncountable Space

Is it compact?

- Yes

20. Countable Complement Topology

Definition:

- Let X be an uncountable set. Open sets are those with countable complements, together with $\varnothing$ (and X)

20. Countable Complement Topolog

Is it compact?

- No

21. Double Pointed Countable Complement Topology

Definition:

- Product of X with the two-point indiscrete space, where X has the countable complement topology as above.

21. Double Pointed Countable Complement Topology

Is it compact?

- No

22. Compact Complement Topology

Definition:

- On $\mathbb{R}$, we define a topology by taking S open whenever either $S = \varnothing$, or $\mathbb{R} \setminus S$ is compact in the usual topology on $\mathbb{R}$.

22. Compact Complement Topology

Is it compact?

- Yes

23. Countable Fort Space

Definition:

- A subset of X is open iff it its complement is finite or includes p.

23. Countable Fort Space

Is it compact?

- Yes

24. Uncountable Fort Space

Definition:

- A subset of X is open iff its complement is finite or includes p.

24. Uncountable Fort Space

Is it compact?

- Yes

25. Fortissimo Space

Definition:

- X is uncountable, and a subset of X is open iff its complement is countable or includes p.

25. Fortissimo Space

Is it compact?

- No

26. Arens-Fort Space

Definition:

- X is the set of ordered pairs of nonnegative integers with each pair open except $(0,0)$. Open neighborhoods U of $(0,0)$ are defined so that, for all but a finite number of integers m, the sets $S_m = \{n : (m,n) \in U\}$ each contain all but a finite number of integers.

26. Arens-Fort Space

Is it compact?

- No

27. Modified Fort Space

Definition:

- X is the union of any infinite set N with two distinct one-point sets $\{x_1\}$ and $\{x_2\}$. Then any subset of N is open, and any subset containing x_1 or x_2 is open iff it contains all but a finite number of elements of N.

27. Modified Fort Space

Is it compact?

- Yes

28. Euclidean Topology

Definition:

- $X = \mathbb{R}$ with basis (a, b) for $a < b$.

28. Euclidean Topology

Is it compact?

- No

29. The Cantor Set

Definition:

- X is all points in $[0, 1]$ which can be expressed in base 3 without using the digit 1, with the subspace topology from $\mathbb{R}$.

29. The Cantor Set

Is it compact?

- Yes

30. The Rational Numbers

Definition:

- The set of rational numbers as a subset of $\mathbb{R}$.

30. The Rational Numbers

Is it compact?

- No

31. The Irrational Numbers

Definition:

- The set of irrational numbers as a subset of $\mathbb{R}$.

31. The Irrational Numbers

Is it compact?

- No

32. Special Subsets Of The Real Line

Definition:

- $A = \{1/n : n = 1, 2, 3, \dots\}$
- $B = \{0\} \cup \{1/n : n = 1, 2, 3, \dots\}$
- $C = (0, 1/2) \cup (1/2, 1)$
- $D = \{1/n : n = 1, 2, \dots\} \cup (2, 3) \cup (3, 4) \cup \{4.5\} \cup [5, 6] \cup \{x :$ x is rational and $7 \leq x < 8\}$

32. Special Subsets Of The Real Line

Is it compact?

- A is no
- B is yes
- C is no
- D is no

33. Special Subsets Of The Plane

Definition:

- $A = \{(x, y) : xy \geq 1\}$
- B is the set of points with at least one irrational coordinate

33. Special Subsets Of The Plane

Is it compact?

- No

34. One Point Compactification Topology

Definition:

- Let (X, τ) be a nonempty topological space and let p be a point not in X. Let $X^* = X \cup \{p\}$ and say that a subset of X^* is open iff it is in τ or it is the complement of a closed and compact subset of (X, τ)

34. One Point Compactification Topology

Is it compact?

- Yes

35. One Point Compactification Of The Rationals

Definition:

- Same as above but with $X = \mathbb{Q}$.

35. One Point Compactification Of The Rationals

Is it compact?

- Yes

36. Hilbert Space

Definition:

- X is the set of sequences of real numbers (x_i) such that $\sum x_i^2$ converges, with the metric topology given by $d(x, y) = \left(\sum(x_i - y_i)^2\right)^{1/2}$

36. Hilbert Space

Is it compact?

- No

37. Fréchet Space

Definition:

- X is the set of sequences of real numbers (x_i) such that $\sum x_i^2$ converges, with the metric topology given by $d(x, y) = \frac{2^{-i}|x_i - y_i|}{1+|x_i - y_i|}$

37. Fréchet Space

Is it compact?

- No

38. Hilbert Cube

Definition:

- Let I^ω denote the set of sequences of elements of $I = [0, 1]$, with the product topology. Then X is the subspace consisting of elements (x_i) with $x_i \le 1/i$ for each i

38. Hilbert Cube

Is it compact?

- Yes

39. Order Topology

Definition:

- If X is any set with a linear order, then we get a topology by taking the open intervals as basis elements

39. Order Topology

Is it compact?

- N/A

40. Open Ordinal Space $[0, \Gamma), \Gamma < \Omega$

Definition:

- X is the set of all ordinals less than some limit ordinal Γ, with $\Gamma < \Omega$, where Ω is the first uncountable ordinal, with the order topology

40. Open Ordinal Space $[0, \Gamma), \Gamma < \Omega$

Is it compact?

- No

41. Closed Ordinal Space $[0, \Gamma], \Gamma < \Omega$

Definition:

- X is the set of all ordinals less than or equal to some limit ordinal Γ, with $\Gamma < \Omega$, where Ω is the first uncountable ordinal, with the order topology

41. Closed Ordinal Space $[0, \Gamma], \Gamma < \Omega$

Is it compact?

- Yes

42. Open Ordinal Space $[0, \Omega)$

Definition:

- X is the set of all ordinals less than Ω, the first uncountable ordinal, with the order topology

42. Open Ordinal Space $[0, \Omega)$

Is it compact?

- No

43. Closed Ordinal Space $[0, \Omega]$

Definition:

- X is the set of all ordinals less than or equal to Ω, the first uncountable ordinal, with the order topology

43. Closed Ordinal Space $[0, \Omega]$

Is it compact?

- Yes

44. Uncountable Discrete Ordinal Space

Definition:

- X is the set of points $\alpha + 1$ in $[0, \Omega)$, where α is a limit ordinal, with the subspace topology from $[0, \Omega)$

44. Uncountable Discrete Ordinal Space

Is it compact?

- No

45. The Long Line

Definition:

- X is constructed from the order space $[0, \Omega)$ by placing between each ordinal α and its successor $\alpha + 1$ a copy of the unit interval $(0, 1)$, and we give X the order topology

45. The Long Line

Is it compact?

- No

46. The Extended Long Line

Definition:

- X is constructed from the order space $[0, \Omega]$ by placing between each ordinal α and its successor $\alpha + 1$ a copy of the unit interval $(0, 1)$, and we give X the order topology

46. The Extended Long Line

Is it compact?

- Yes

47. An Altered Long Line

Definition:

- To the long line L, we add a point p. Open sets are the open sets of L, together with those generated by $U_\beta(p) = \{p\} \cup \{\bigcup_{\alpha=\beta}^{\Omega}(\alpha, \alpha+1)\}$ (where $1 \leq \beta < \Omega$)

47. An Altered Long Line

Is it compact?

- No

48. Lexicographic Ordering On The Unit Square

Definition:

- We say $(x, y) < (u, v)$ when either $x < u$ or $x = u$ and $y < v$, and give the unit square the order topology

48. Lexicographic Ordering On The Unit Square

Is it compact?

- Yes

49. Right Order Topology

Definition:

- If X is a linearly ordered set, we take the topology generated by $S_a = \{x \in X : x > a\}$

49. Right Order Topology

Is it compact?

- N/A

50. Right Order Topology on $\mathbb{R}$

Definition:

- $X = \mathbb{R}$, and we take the topology generated by $S_a = \{x \in X : x > a\}$

50. Right Order Topology on $\mathbb{R}$

Is it compact?

- No

51. Right Half-Open Interval Topology

Definition:

- $X = \mathbb{R}$, and we take the topology generated by $\{[a, b)\}$

51. Right Half-Open Interval Topology

Is it compact?

- No

52. Nested Interval Topology

Definition:

- $X = (0, 1)$, and the open sets are $(0, 1 - 1/n)$, for $n = 2, 3, 4, \ldots$, along with $\varnothing$ and X

52. Nested Interval Topology

Is it compact?

- No

53. Overlapping Interval Topology

Definition:

- $X = [-1, 1]$, and the open sets are generated by $[-1, b)$ for $b > 0$ and $(a, 1]$ for $a < 0$

53. Overlapping Interval Topology

Is it compact?

- Yes

54. Interlocking Interval Topology

Definition:

- $X = \mathbb{R}^+ \setminus \mathbb{Z}^+$ and the topology is generated by $(0, 1/n) \cup (n, n+1)$

54. Interlocking Interval Topology

Is it compact?

- No

55. Hjalmar Ekdal Topology

Definition:

- $X = \mathbb{Z}$, and a set U is open iff, for every odd integer n in U, the integer $n + 1$ is in U

55. Hjalmar Ekdal Topology

Is it compact?

- No

56. Prime Ideal Topology

Definition:

- X is the set of prime ideals of $\mathbb{Z}$, and take as a basis for the topology all sets $V_x = \{P \in X : x \notin P\}$

56. Prime Ideal Topology

Is it compact?

- Yes

57. Divisor Topology

Definition:

- $X = \{x \in \mathbb{Z} : x \geq 2\}$, and open sets are generated by $U_n = \{x \in X : x \text{ divides } n\}$

57. Divisor Topology

Is it compact?

- No

58. Evenly Spaced Integer Topology

Definition:

- $X = \mathbb{Z}$, and open sets are generated by $a + k\mathbb{Z}$, $a, k \in \mathbb{Z}$

58. Evenly Spaced Integer Topology

Is it compact?

- No

59. The p-adic Topology on $\mathbb{Z}$

Definition:

- $X = \mathbb{Z}$, p is a fixed prime, and we take as a basis the sets of the form $U_\alpha(n) = \{n + \lambda p^\alpha : \lambda \in \mathbb{Z}\}$

59. The p-adic Topology on $\mathbb{Z}$

Is it compact?

- No

60. Relatively Prime Integer Topology

Definition:

- X is the set of positive integers, and we generate a topology from the basis $\{U_a(b) : (a, b) = 1\}$

60. Relatively Prime Integer Topology

Is it compact?

- No

61. Prime Integer Topology

Definition:

- X is the set of positive integers, and we generate a topology from the basis $\{\, U_p(b) : p \text{ prime } \}$

61. Prime Integer Topology

Is it compact?

- No

62. Double Pointed Reals

Definition:

- X is the product of $\mathbb{R}$ with the usual topology and $\{0, 1\}$ with the indiscrete topology

62. Double Pointed Reals

Is it compact?

- No

63. Countable Complement Extension Topology

Definition:

- $X = \mathbb{R}$, let τ_1 be the usual topology, and let τ_2 be the topology of countable complements, and we let τ be the smallest topology generated by $\tau_1 \cup \tau_2$

63. Countable Complement Extension Topology

Is it compact?

- No

64. Smirnov's Deleted Sequence Topology

Definition:

- $X = \mathbb{R}$ and let $A = \{1/n : n = 1, 2, \ldots\}$, and a set O is open if it is equal to $U \setminus B$, for some $B \subseteq A$

64. Smirnov's Deleted Sequence Topology

Is it compact?

- No

65. Rational Sequence Topology

Definition:

- $X = \mathbb{R}$, every rational singleton is open, and for each irrational x, we choose a sequence (x_i) of rationals converging to x, and then also the sets $U_n(x) = \{x_i\}_{i=n}^{\infty} \cup \{x\}$ form a local basis at each irrational point

65. Rational Sequence Topology

Is it compact?

- No

66. Indiscrete Rational Extension Of $\mathbb{R}$

Definition:

- $X = \mathbb{R}$, topology generated by the usual sets, plus all sets of the form $\mathbb{Q} \cap U$, where U is a usual open set

66. Indiscrete Rational Extension Of $\mathbb{R}$

Is it compact?

- No

67. Indiscrete Irrational Extension Of $\mathbb{R}$

Definition:

- $X = \mathbb{R}$, topology generated by the usual sets, plus all sets of the form $(\mathbb{R} \setminus \mathbb{Q}) \cap U$, where U is a usual open set

67. Indiscrete Irrational Extension Of $\mathbb{R}$

Is it compact?

- No

68. Pointed Rational Extension Of $\mathbb{R}$

Definition:

- $X = \mathbb{R}$, topology generated by the usual sets, plus all sets of the form $\{x\} \cup (\mathbb{Q} \cap U)$, where U is a usual open set and $x \in U$

68. Pointed Rational Extension Of $\mathbb{R}$

Is it compact?

- No

69. Pointed Irrational Extension Of $\mathbb{R}$

Definition:

- $X = \mathbb{R}$, topology generated by the usual sets, plus all sets of the form $\{x\} \cup ((\mathbb{R} \setminus \mathbb{Q}) \cap U)$, where U is a usual open set and $x \in U$

69. Pointed Irrational Extension Of $\mathbb{R}$

Is it compact?

- No

70. Discrete Rational Extension Of $\mathbb{R}$

Definition:

- $X = \mathbb{R}$, topology generated by the usual sets, plus all rational singletons are open

70. Discrete Rational Extension Of $\mathbb{R}$

Is it compact?

- No

71. Discrete Irrational Extension Of $\mathbb{R}$

Definition:

- $X = \mathbb{R}$, topology generated by the usual sets, plus all irrational singletons are open

71. Discrete Irrational Extension Of $\mathbb{R}$

Is it compact?

- No

72. Rational Extension In The Plane

Definition:

- $X = \mathbb{R}^2$, each point in the set $D = \{(x, y) : x, y \in \mathbb{Q}\}$ is open, and each set of the form $\{x\} \cup (D \cap U)$ is open, where U is open in the usual topology, and $x \in U$

72. Rational Extension In The Plane

Is it compact?

- No

73. Telophase Topology

Definition:

- Start with $[0, 1]$ and add another right endpoint called 1^*, and then the usual sets are open, plus the sets $(a, 1) \cup \{1^*\}$ form a local basis of 1^*

73. Telophase Topology

Is it compact?

- Yes

74. Double Origin Topology

Definition:

- $X = \mathbb{R}^2 \cup \{0^*\}$, and neighborhoods of points other than 0 and 0^* are the usual open sets of $\mathbb{R}^2 \setminus \{0\}$. As a basis around the point $\{0\}$ we take the sets $V_n(0) = \{(x, y) : x^2 + y^2 < 1/n^2, y > 0\} \cup \{0\}$ and as a basis around the point $\{0^*\}$ we take the sets $V_n(0^*) = \{(x, y) : x^2 + y^2 < 1/n^2, y < 0\} \cup \{0^*\}$

74. Double Origin Topology

Is it compact?

- No

75. Irrational Slope Topology

Definition:

- $X = \{(x, y) \in \mathbb{R}^2 : y \geq 0, (x, y) \in \mathbb{Q}\}$, fix an irrational number θ. Topology generated by sets of the form $N_\epsilon((x, y)) = \{(x, y)\} \cup B_\epsilon(x + y/\theta) \cup B_\epsilon(x - y/\theta)$, where $B_\epsilon(\zeta)$ denotes the epsilon neighborhood of $\mathbb{Q}$ as a subset of the x-axis.

75. Irrational Slope Topology

Is it compact?

- No

76. Deleted Diameter Topology

Definition:

- $X = \mathbb{R}^2$, and we take as a subbasis the open discs with horizontal punctured diameter deleted

76. Deleted Diameter Topology

Is it compact?

- No

77. Deleted Radius Topology

Definition:

- $X = \mathbb{R}^2$, and we take as a subbasis the open discs with right open horizontal radius deleted

77. Deleted Radius Topology

Is it compact?

- No

78. Half-Disc Topology

Definition:

- $X = \{(x, y) \in \mathbb{R}^2 : y > 0\}$, and if L is the x-axis, we generate a topology on $X \cup L$ by adding all sets of the form $\{x\} \cup (X \cap U)$, where $x \in L$ and U is a usual neighborhood of x in the plane

78. Half-Disc Topology

Is it compact?

- No

79. Irregular Lattice Topology

Definition:

- X is the subset of $\mathbb{Z}^2$ consisting of points (x, y) with $x > 0$ and $y > 0$, and $(x, 0)$, with $x \geq 0$. Each of the former points is open. Each point of the form $(i, 0)$, $i \neq 0$ has a local basis sets of the form $U_n((i, 0)) = \{(i, k) : k = 0 \text{ or } k \geq n\}$, and the point $(0, 0)$ has local basis $V_n = \{(i, k) : i = k = 0 \text{ or } i, k \geq n\}$

79. Irregular Lattice Topology

Is it compact?

- No

80. Arens Square

Definition:

- Let $S = \{(x, y) \in (0,1) \times (0,1) : x, y \in \mathbb{Q}\}$ and $X = S \cup \{(0,0)\} \cup \{(1,0)\} \cup \{(1/2, r\sqrt{2}) : r \in \mathbb{Q}, 0 < r\sqrt{2} < 1\}$ Each point of S has local basis from the subspace topology on $\mathbb{R}^2$. The other points have the following local bases:
- $U_n(0,0) = \{(0,0)\} \cup \{(x, y) : 0 < x < 1/4, 0 < y < 1/n\}$
- $U_n(1,0) = \{(1,0)\} \cup \{(x, y) : 3/4 < x < 1, 0 < y < 1/n\}$
- $U_n(1/2, r\sqrt{2}) = \{(x, y) : 1/4 < x < 3/4, |y - r\sqrt{2}| < 1/n\}$

80. Arens Square

Is it compact?

- No

81. Simplified Arens Square

Definition:

- Let S be the set of points in the interior of the unit square, and let $X = S \cup \{(0,0),(1,0)\}$. Points in S are given local bases from the subspace topology on $\mathbb{R}^2$, and the other points have local bases:
- $U_n(0,0) = \{(0,0)\} \cup \{(x,y) : 0 < x < 1/2, 0 < y < 1/n\}$
- $U_m(1,0) = \{(1,0)\} \cup \{(x,y) : 1/2 < x < 1, 0 < y < 1/m\}$

81. Simplified Arens Square

Is it compact?

- No

82. Niemytzki's Tangent Disc Topology

Definition:

- Let $P = \{(x, y) \in \mathbb{R}^2 : y > 0\}$ and L denote the x-axis. Then $X = P \cup L$, and we take the usual topology and add in all sets of the form $\{x\} \cup D$, where D is an open disc tangent to L at x

82. Niemytzki's Tangent Disc Topology

Is it compact?

- No

83. Metrizable Tangent Disc Topology

Definition:

- Let S be a countable subset of the x-axis in the plane. We take the subspace of the tangent disc topology consisting of $P \cup S$

83. Metrizable Tangent Disc Topology

Is it compact?

- No

84. Sorgenfrey's Half-Open Square Topology

Definition:

- Let S denote the real line with the right half-open interval topology. Then $X = S \times S$ with the product topology

84. Sorgenfrey's Half-Open Square Topology

Is it compact?

- No

85. Michael's Product Topology

Definition:

- Let $(\mathbb{R}, \tau)$ denote the real line with the usual topology. Let $D = \mathbb{R} \setminus \mathbb{Q}$. Then X is the product space $(\mathbb{R}, \tau^*) \times (D, \tau')$, where τ^* is the irrational discrete extension of τ by D, and τ' is the subspace topology from the usual topology on $\mathbb{R}$

85. Michael's Product Topology

Is it compact?

- No

86. Tychonoff Plank

Definition:

- If Ω is the first uncountable ordinal and ω is the first countable ordinal, then X is the product $[0, \Omega] \times [0, \omega]$, where both spaces are given the interval topology.

86. Tychonoff Plank

Is it compact?

- Yes

87. Deleted Tychonoff Plank

Definition:

- If Ω is the first uncountable ordinal and ω is the first countable ordinal, then X is the product $([0,\Omega] \times [0,\omega]) \setminus \{(\Omega,\omega)\}$, where both spaces are given the interval topology.

87. Deleted Tychonoff Plank

Is it compact?

- No

88. Alexandroff Plank

Definition:

- X is the product $[0,\Omega] \times [-1,1]$, each with the interval topology, and then we take the expansion generated by adding all sets of the form $U(\alpha,n) = \{(\Omega,0)\} \cup (\alpha,\Omega] \times (0,1/n)$

88. Alexandroff Plank

Is it compact?

- No

89. Dieudonné Plank

Definition:

- $X = [0, \Omega] \times [0, \omega] \setminus \{(\Omega, \omega)\}$, and the open sets are each point of $[0, \Omega) \times [0, \omega)$, along with $U_\alpha(\beta) = \{(\beta, \gamma) : \alpha < \gamma \leq \omega\}$ and $V_\alpha(\beta) = \{(\gamma, \beta) : \alpha < \gamma \leq \Omega\}$

89. Dieudonné Plank

Is it compact?

- No

90. Tychonoff Corkscrew

Definition:

- For each ordinal α, let A_α denote the linearly ordered set $(-0, -1, \ldots, \alpha, \ldots, 2, 1, 0)$ with the order topology. Let P denote the product space $A_\Omega \times A_\omega$. Let P^* be the subsapce $P \setminus \{(\Omega, \omega)\}$
- Then take an infinite stack of copies of P^* and cut each of these planes just below the positive A_Ω-axis (I don't know what this means), and join the fourth quadrant of each plane to the first quadrant of the one immediately below it, and denote this by S
- Then add two points a^+ and a^-, "infinity points" at the top and bottom of the corkscrew, with open neighborhoods given by all points of the corkscrew which lie above a certain level (or below, for a^-)

90. Tychonoff Corkscrew

Is it compact?

- No

91. Deleted Tychonoff Corkscrew

Definition:

- Take the Tychonoff Corkscrew and delete $\{a^-\}$

91. Deleted Tychonoff Corkscrew

Is it compact?

- No

92. Hewitt's Condensed Corkscrew

Definition:

- Let $T = S \cup \{a^+\} \cup \{a^-\}$ denote the Tychonoff Corkscrew, and if $[0, \Omega)$ is the set of countable ordinals, we let $A = T \times [0, \Omega)$, and let X be the subset of A consisting of $S \times [0, \Omega)$
- We think of A as an uncountable sequence of corkscrews A_λ, with $\lambda \in [0, \Omega)$, and X as the same sequence of corkscrews missing all infinity points
- If $\Gamma : X \times X \to [0, \Omega)$ is a bijection, and if π_i, $(i = 1, 2)$ are the coordinate projections $X \times X \to X$, then we define a function $\psi : A \setminus X \to X$ by $\psi(a_\lambda^+) = \pi_1(\Gamma^{-1}(\lambda))$ and $\psi(a_\lambda^-) = \pi_2(\Gamma^{-1}(\lambda))$
- Basis neighborhoods of A are subsets N of A with the property that $\psi^{-1}(X \cap N) \subseteq N$, along with A_λ-basis neighborhoods of each point $a \in A \setminus X$
- X gets the subspace topology from A

92. Hewitt's Condensed Corkscrew

Is it compact?

- No

93. Thomas's Plank

Definition:

- $X = \bigcup_{i=0}^{\infty} L_i$ of lines in the plane, where $L_0 = \{(x, 0) : x \in (0, 1)\}$ and for $i \geq 1$, $L_i = \{(x, 1/i) : x \in [0, 1)\}$.
- If $i \geq 1$, each point of L_i except for $(0, 1/i)$ is open.
- Basis neighborhoods of $(0, 1/i)$ are subsets of L_i with finite complements
- The sets $U_i(x, 0) = \{(x, 0)\} \cup \{(x, 1/n) : n > i\}$ form a basis for the points in L_0

93. Thomas's Plank

Is it compact?

- No

94. Thomas's Corkscrew

Definition:

- Take an infinite stack of Thomas's Planks to build a corkscrew, as in the Tychonoff Corkscrew (the book is literally this vague)

94. Thomas's Corkscrew

Is it compact?

- No

95. Weak Parallel Line Topology

Definition:

- Let A be the subset of the plane $\{(x,0) : 0 < x \leq 1\}$, and let B be the subset $\{(x,1) : 0 \leq x < 1\}$
- X is the set $A \cup B$
- Take as a basis sets of the form $\{(x,0) : a < x \leq b\} \cup \{(x,1) : a < x < b\}$ and $\{(x,0) : a < x < b\} \cup \{(x,1) : a \leq x < b\}$

95. Weak Parallel Line Topology

Is it compact?

- No

96. Strong Parallel Line Topology

Definition:

- Same set as above, but we take as a basis all sets of the form $\{(x,1) : a \leq x < b\}$ and $\{(x,0) : a < x \leq b\} \cup \{(x,1) : a < x < b\}$

96. Strong Parallel Line Topology

Is it compact?

- No

97. Concentric Circles

Definition:

- X consists of two concentric circles, C_1 the inner circle and C_2 the outer circle
- Take as a subbasis all singleton sets in C_2, and all open intervals in C_1, each together with the radial projection of all but its midpoint on C_2

97. Concentric Circles

Is it compact?

- Yes

98. Appert Space

Definition:

- X is the set of positive integers, and let $N(n, E)$ denote the number of integers in a subset $E \subseteq X$ which are less than or equal to n
- Open sets are any set which excludes the integer 1, or any set containing 1 and for which $\lim_{n\to\infty} N(n, E)/n = 1$

98. Appert Space

Is it compact?

- No

99. Maximal Compact Topology

Definition:

- X is the set of all lattice points of positive integers (i, j), together with two ideal points x and y
- Each lattice point is open, and open neighborhoods of x are sets of the form $X \setminus A$, where A is any subset of X with at most finitely many points in each row
- Open neighborhoods of y are sets of the form $X \setminus B$, where B is any subset of X consisting of points from at most finitely many rows

99. Maximal Compact Topology

Is it compact?

- Yes

100. Minimal Hausdorff Topology

Definition:

- If A is the linearly ordered set $(1, 2, \ldots, \omega, \ldots, -3, -2, -1)$ with the interval topology , and if $\mathbb{Z}^+$ is the set of positive integers with the discrete topology, then we define X to be $A \times \mathbb{Z}^+$, together with two ideal points a and $-a$
- Topology is the product topology on $A \times \mathbb{Z}^+$, as well as basis neighborhoods $M_n(a) = \{a\} \cup \{(i, j) : i < \omega, j > n\}$ and $M_n(-a) = \{-a\} \cup \{(i, j), i > \omega, j > n\}$

100. Minimal Hausdorff Topology

Is it compact?

- No

101. Alexandroff Square

Definition:

- X is the closed unit square $[0,1] \times [0,1]$
- For points (s,t) off the diagonal, we take as a local basis the collection of open vertical line segments which do not intersect the diagonal
- Neighborhoods of points on the diagonal are open horizontal strips, minus a finite number of vertical line segments

101. Alexandroff Square

Is it compact?

- Yes

102. $\mathbb{Z}^{\mathbb{Z}}$

Definition:

- Let $\mathbb{Z}^+$ have the discrete topology, and let $X = \prod_{i=1}^{\infty} Z^+$ be the countably infinite cartesian product, with the product topology

102. $\mathbb{Z}^{\mathbb{Z}}$

Is it compact?

- No

103. Uncountable Products Of $\mathbb{Z}^+$

Definition:

- Let $\mathbb{Z}^+$ have the discrete topology, and let $X = \prod_{a \in A} Z_a^+$ be the Cartesian product, with the product topology, where A is uncountable

103. Uncountable Products Of $\mathbb{Z}^+$

Is it compact?

- No

104. Baire Metric on $\mathbb{R}^\omega$

Definition:

- $X = \mathbb{R}^\omega = \prod_{i=1}^{\infty} \mathbb{R}_i$, where each $\mathbb{R}_i$ is a copy of the real line
- We define a metric $d((x_i), (y_i)) = 1/i$, where i is the first coordinate in which (x_i) and (y_i) differ

104. Baire Metric on $\mathbb{R}^\omega$

Is it compact?

- No

105. I^I

Definition:

- X is the uncountable Cartesian product of $[0, 1]$ with itself

105. I^I

Is it compact?

- Yes

106. $[0, \Omega) \times I^I$

Definition:

- X is the product of $[0, \Omega)$ with the interval topology and I^I with the product topology

106. $[0, \Omega) \times I^I$

Is it compact?

- No

107. Helly Space

Definition:

- Subspace of I^I consisting of all nondecreasing functions

107. Helly Space

Is it compact?

- Yes

108. $C[0, 1]$

Definition:

- Space of real-valued continuous functions on the unit interval, with metric given by $d(f, g) = \sup_{t \in I}(f(t) - g(t))$

108. $C[0, 1]$

Is it compact?

- No

109. Boolean Product Topology On $\mathbb{R}^\omega$

Definition:

- $X = \mathbb{R}^\omega$, and open sets are generated by $\prod_{i=1}^{\infty} U_i$, where each U_i is open in $\mathbb{R}$

109. Boolean Product Topology On $\mathbb{R}^\omega$

Is it compact?

- No

110. Stone-Čech Compactification

Definition:

- Let (X, τ) be a completely regular space, let I be the closed unit interval $[0, 1] \subseteq \mathbb{R}$, and let $C(X, I)$ be the collection of all continuous functions from X to I
- Let $I^{C(X,I)} = \prod_{\lambda \in C(X,I)} I_\lambda$, where I_λ is a copy of I (indexed by λ)
- Denote by $\langle t_\lambda \rangle$ the element of $I^{C(X,I)}$ whose λ coordinate is t_λ
- If $h_X : X \to I^{C(X,I)}$ is defined by $h_X(x) = \langle \lambda(x)_\lambda \rangle$, the image of h_x is a subset of $I^{C(X,I)}$
- Then we take its closure and denote it βX

110. Stone-Čech Compactification

Is it compact?

- Yes

111. Stone-Čech Compactification Of The Integers

Definition:

- X the Stone-Čech compactification of $\mathbb{Z}^+$, the space of positive integers with the discrete topology

111. Stone-Čech Compactification Of The Integers

Is it compact?

- Yes

112. Novak Space

Definition:

- Let $\mathbb{Z}^+$ denote the space of positive integers with the discrete topology, and S is the Stone-Čech compactification of $\mathbb{Z}^+$
- Let F be the family of all countably infinite subsets of S, and well-order this set (it has cardinality 2^c)
- Let $\{P_A : A \in F\}$ be a collection of subsets of S such that card$(P_A) < 2^c$, $P_D \subseteq P_A$ whenever $D < A$, and $\bar{f}(P_A) \cap P_A = \varnothing$, where $\bar{f} : S \to S$ is the unique continuous extension of the function $f : \mathbb{Z} \to \mathbb{Z}$ which interchanges each odd number with its even successor
- Then let $P = \bigcup\{P_A : A \in F\}$ and let $X = P \cup \mathbb{Z}^+$

112. Novak Space

Is it compact?

- No

113. Strong Ultrafilter Topology

Definition:

- Let $\mathbb{Z}^+$ be the positive integers, and let M denote the collection of all non-principal ultrafilters on $\mathbb{Z}^+$
- Let $X = \mathbb{Z}^+ \cup M$, with topology generated by the points of $\mathbb{Z}^+$, together with all sets of the form $A \cup \{F\}$, where $A \in F \in M$

113. Strong Ultrafilter Topology

Is it compact?

- No

114. Single Ultrafilter Topology

Definition:

- Let $X = \mathbb{Z}^+ \cup \{F\}$, where F is a non-principal ultrafilter on $\mathbb{Z}^+$, with basis all points of $\mathbb{Z}^+$, together with all subsets of the form $A \cup F$, where $A \in F$

114. Single Ultrafilter Topology

Is it compact?

- No

115. Nested Rectangles

Definition:

- In $\mathbb{R}^2$, let L_1 denote the line $x = 1$, L_2 denote the line $x = -1$, and R_n the boundary of rectangles centered at the origin, of height $2n$ and width $2n/(n+1)$, and let $X = L_1 \cup L_2 \cup (\bigcup R_n)$, with subspace topology from $\mathbb{R}^2$

115. Nested Rectangles

Is it compact?

- No

116. Topologist's Sine Curve

Definition:

- Let S be the graph of $f(x) = \sin(1/x)$ for $0 < x \leq 1$, and then $X = S \cup \{(0,0)\}$ with subspace topology from $\mathbb{R}^2$

116. Topologist's Sine Curve

Is it compact?

- No

117. Closed Topologist's Sine Curve

Definition:

- Let S be as above, and then $X = S \cup \{(0, y) : -1 \leq y \leq 1\}$

117. Closed Topologist's Sine Curve

Is it compact?

- Yes

118. Extended Topologist's Sine Curve

Definition:

- Take the closed topologists sine curve and add $\{(x, 1) : 0 \leq x \leq 1\}$

118. Extended Topologist's Sine Curve

Is it compact?

- Yes

119. The Infinite Broom

Definition:

- X is the union of the closed line segments joining the origin to the points $\{(1, 1/n) : n = 1, 2, 3, \ldots\}$, together with the half-open interval $(1/2, 1]$ on the x-axis

119. The Infinite Broom

Is it compact?

- No

120. The Closed Infinite Broom

Definition:

- The closure of the infinite broom, so the union of the broom with $(0, 1]$

120. The Closed Infinite Broom

Is it compact?

- Yes

121. The Integer Broom

Definition:

- X is the set of points with polar coordinates $\{(n, \theta)\}$ in the plane, where n is a nonnegative integer and $\theta \in \{1/n\}_{n=1}^{\infty} \cup \{0\}$
- Take as a basis all sets of the form $U \times V$, where U is an open set in the right-order topology on the set of nonnegative integers, and V is open in $\{0\} \cup \{1/n\}_{n=1}^{\infty}$ in the topology induced from the reals. The only neighborhood of the origin is X itself

121. The Integer Broom

Is it compact?

- Yes

122. Nested Angles

Definition:

- X is the subset of the plane consisting of line segments joining the points $(0,1)$ and $(n, 1/(n+1))$, for $n \in \mathbb{Z}^+$, and the half-lines $y = 1/(n+1)$, when $x \leq n$, and the line $y = 0$

122. Nested Angles

Is it compact?

- No

123. The Infinite Cage

Definition:

- $X = \bigcup_{i=1}^{n} A_n \cup B_n \cup C_n$ is the union of three types of sets:
- $A_n = \{(1/n, y, 0) \in \mathbb{R}^3 : y \geq 0\}$
- $B_n = \{(0, y, 0) \in \mathbb{R}^3 : 2n - 1/2 \leq y \leq 2n + 1/2\}$
- $C_n = \{(x, y, z) \in \mathbb{R}^3 : 0 \leq x \leq 1/n, y = 2n, z = x(1/n - x)\}$

123. The Infinite Cage

Is it compact?

- No

124. Bernstein's Connected Sets

Definition:

- Let $\{C_\alpha : \alpha \in [0, \Gamma)\}$ be the collection of all nondegenerate, closed, connected subsets of $\mathbb{R}^2$, well-ordered by Γ, the least ordinal equivalent to c, the cardinal of the continuum
- Define by transfinite induction two nested sequences $\{A_\alpha\}_{\alpha<\Gamma}$ and $\{B_\alpha\}_{\alpha<\Gamma}$ such that $A_\alpha \cap B_\beta = \varnothing$ for all pairs α, β
- A_1 and B_1 are distinct singletons selected from C_1, and if $\{A_\alpha\}_{\alpha<\beta}$ and $\{B_\alpha\}_{\alpha<\beta}$ have been defined, then the size of $\bigcup_{\alpha<\beta}(A_\alpha \cup B_\alpha)$ is less than c, but the size of C_β is equal to c, so we can choose two points a_β and b_β in $C_\beta \setminus \bigcup_{\alpha<\beta}(A_\alpha \cup B_\alpha)$ and add them to our sets to get A_β and B_β
- Then we let A be the union of all the A_α and $B = \mathbb{R}^2 \setminus A$

124. Bernstein's Connected Sets

Is it compact?

- N/A

125. Gustin's Sequence Space

Definition:

- Let Y be the collection of all finite sequences of positive integers having an even number of terms, including the null sequence denoted by 0
- Let W be the collection of all subsets of size two of Y
- The set X is $Y \cup (\mathbb{Z}^+ \times W)$
- If α and β are arbitrary finite sequences, we denote by $\alpha\beta$ the sequence you get by adjoining β to the end of α
- We say that $\alpha \geq i$ if $a \geq i$ for each $a \in \alpha$
- We say $\beta \supseteq_i \alpha$ if there exists a sequence $\gamma \geq i$ such that $\beta = \alpha\gamma$
- For a sequence α, let $U_i(\alpha) = \{\beta \in Y : \beta \supseteq_i \alpha\}$
- Now select a bijection between the countable set W and the set of positive prime numbers
- Define $q : (\mathbb{Z}^+ \times W) \to \mathbb{Z}^+$ by $q(n, w) = p(w)^n$, where $p(w)$ is the prime corresponding to w
- Define the topology on X by selecting $U_i(\alpha)$ as open neighborhoods of the point $\alpha \in Y$, when $w = \{\alpha, \beta\}$, let $V_i(n, w) = \{(n, w)\} \cup U_i(\alpha q(n, w)) \cup U_i(\beta q(n, w))$ be the open neighborhoods of (n, w)

125. Gustin's Sequence Space

Is it compact?

- No

126. Roy's Lattice Space

Definition:

- Let $\{C_i\}_{i=1}^{\infty}$ be a countable collection of disjoint dense subsets of $\mathbb{Q}$
- Let X be the set $\{(r,i) \in \mathbb{Q} \times \mathbb{Z}^+ : r \in C_i\}$, together with an ideal point ω
- Neighborhoods of points of the form $(r, 2n)$ are ordinary open intervals $U_\epsilon(r,2n) = \{(t,2n) : |t-r| < \epsilon\}$
- Neighborhoods of points of the form $(r, 2n-1)$ are stacks of three open intervals $V_\epsilon(r,2n-1) = \{(t,m) : |t-r| < \epsilon, m = 2n-2, 2n-1, 2n\}$
- A basis neighborhood of the point ω consists of $\{(s,i) \in X : i \geq 2n\}$

126. Roy's Lattice Space

Is it compact?

- No

127. Roy's Lattice Subspace

Definition:

- Start with Roy's Lattice Space and delete the point ω

127. Roy's Lattice Subspace

Is it compact?

- No

128. Cantor's Leaky Tent

Definition:

- Let C be the Cantor set on the unit interval $[0, 1]$
- Let p be the point $(1/2, 1/2)$ in $\mathbb{R}^2$
- Let X be the cone over C with vertex at p (the union of line segments joining p to the points in C)

128. Cantor's Leaky Tent

Is it compact?

- No

129. Cantor's Teepee

Definition:

- Let E denote the subset of C consisting of endpoints of deleted intervals
- Let X_E denote the cone over E
- Let $F = C \setminus E$, and let X_F denote the cone over F
- Let $Y_E = \{(x, y) \in X_E : y \in \mathbb{Q}\}$
- Let $Y_F = \{(x, y) \in X_F : y \notin \mathbb{Q}\}$
- Our space is $Y = Y_E \cup Y_F$

129. Cantor's Teepee

Is it compact?

- No

130. A Pseudo-Arc

Definition:

- We define a chain $\mathcal{D}$ in $\mathbb{R}^2$ to be a finite collection of open sets $\{D_i\}_{i=1}^n$ (called links) such that $D_i \cap D_j = \varnothing$ iff $|i-j| > 1$
- A pseudo-arc joining two points $a, b \in \mathbb{R}^2$ is any set in $\mathbb{R}^2$ resulting from the following construction:
- Let $\{\mathcal{D}_i\}$ be a sequence of chains such that:
- The diameter of each open set in $\mathcal{D}_i$ is less than $1/i$
- The closure of each link of D_{i+1} is contained in some link of $\mathcal{D}_i$
- $\mathcal{D}_{i+1}$ is "crooked" in $\mathcal{D}_i$, i.e. if $D_m^{i+1}, D_n^{i+1} \in \mathcal{D}_{i+1}$ with $m < n$, and $D_m^{i+1} \subseteq D_h^i$, and $D_n^{i+1} \subseteq D_k^i$, with $|k-h| > 2$, then there exist $D_s^{i+1}, D_t^{i+1} \in \mathcal{D}_{i+1}$ with $m < s < t < n$ such that D_s^{i+1} is contained in a link of $\mathcal{D}_i$ adjacent to D_k^i, and D_t^{i+1} is contained in a link adjacent to D_k^i
- There are two points a and b with a in the first link of each chain $\mathcal{D}_i$ and b in the final link of each chain
- Let $\mathcal{D}_i^* = \bigcup_k D_k^i$ denote the set of all elements of elements of $\mathcal{D}_i$, and let $X = \bigcap_i \overline{\mathcal{D}_i^*}$

130. A Pseudo-Arc

Is it compact?

- Yes

131. Miller's Biconnected Set

Definition:

- Let C be a nowhere dense perfect set contained in the unit interval I (i.e. it is closed and has no isolated points and has empty interior)
- Let $W = C \times I \subseteq \mathbb{R}^2$
- Let K be an indecomposable continuum (compact connected) such that $K \cap I^2 = W$
- X is defined using the axiom of choice: Let $\mathcal{C}$ be the set of composants of K (a composant is a maximal subset in which any two points lie within some proper subcontinuum)
- Let $\mathcal{B}$ be the set of continua which separate K
- Let $\mathcal{D}$ be the set of subsets of a fixed countable dense subset Δ of K which are themselved desnse in the interior of some square region with edges parallel to I^2 which intersects W
- Let $C_1, C_2, \ldots C_\alpha$ be a well-ordering of the ordinals less than Ω
- Let $B_1, \ldots$ and $D_1, \ldots$ be well orderings of $\mathcal{B}$ and $\mathcal{D}$
- For each $\alpha < \Omega$, define $M_\alpha \subseteq K$ and a simple closed curve J_α such that:
- $M_\alpha = p_\alpha \in B_\alpha \cap K$ if $B_\alpha \cap \Delta = \varnothing$
- $M_\alpha = \varnothing$ if $B_\alpha \cap \Delta \neq \varnothing$
- For ordinals $\mu \neq \lambda$ and $M_\mu, M_\lambda \neq \varnothing$, M_μ and M_λ belong to different components of K
- J_α separates K
- $J_\alpha \cap (\Delta \setminus D_\alpha) = J_\alpha \cap M = \varnothing$, where $M = \bigcup_{\alpha<\Omega} M_\alpha$
- The space X is $\Delta \cup M$ with the subspace topology from $\mathbb{R}^2$

131. Miller's Biconnected Set

Is it compact?

- I don't know

132. Wheel Without Its Hub

Definition:

- X is the closed unit disc in $\mathbb{R}^2$ minus the origin
- We generate a topology by adding to the usual open sets, all radii contained in the open unit disc

132. Wheel Without Its Hub

Is it compact?

- No

133. Tangora's Connected Space

Definition:

- Let X, Y, Z be mutually disjoint and exhaustive dense subsets of $\mathbb{R}$
- Expand the usual topology on $\mathbb{R}$ by adding as open sets X, Y, and sets of the form $\{z\} \cup \{w \in X \cup Y : |w - z| < \delta\}$ where $z \in Z$ and $\delta > 0$

133. Tangora's Connected Space

Is it compact?

- No

134. Bounded Metrics

Definition:

- If (X, d) is a metric space, we define new metric for X by $\delta = d/(1+d)$ and $\Delta = \min(d, 1)$

134. Bounded Metrics

Is it compact?

- N/A

135. Sierpinski's Metric Space

Definition:

- $X = \{x_i : i = 1, 2, 3, \ldots\}$ is a countable set, and the function $d(x_i, x_j) = 1 + 1/(i + j)$ for $i \neq j$ is a metric on X $(d(x_i, x_i) = 0)$

135. Sierpinski's Metric Space

Is it compact?

- No

136. Duncan's Space

Definition:

- Let $N(n, x)$ be the number of elements of the sequence $x = (x_i)$ which are less than n
- Let X be the set of strictly increasing sequences of positive integers such that $\delta((x_i)) = \lim_{n\to\infty} N(n, x)/n$ exists
- Let $k(x, y)$ be the least integer n for which $x_n \neq y_n$
- Define a metric on X by the condition $d(x, y) = 1/k(x, y) + |\delta(x) - \delta(y)|$, and if $x = y$ then $d(x, y) = 0$

136. Duncan's Space

Is it compact?

- No

137. Cauchy Completion

Definition:

- If (X, d) is a metric space, let X^* be the set of all equivalence classes of Cauchy sequences, where the sequence (x_n) is equivalent to (y_n) if $\lim_{n\to\infty} d(x_n, y_n) = 0$
- Define d^* on X^* by $d^*(x^*, y^*) = \lim_{n\to\infty} d((x_n), (y_n))$, where (x_n) and (y_n) are any elements of the equivalence classes x^* and y^*

137. Cauchy Completion

Is it compact?

- N/A

138. Hausdorff's Metric Topology

Definition:

- Let (S, d) be a metric space, and let X be the collection of all nonempty bounded closed subsets of S
- Let $f: S \times X \to \mathbb{R}^+$ be defined by $f(s, B) = \inf_{b \in B} d(s, b)$
- Let $g: X \times X \to \mathbb{R}^+$ be given by $g(A, B) = \sup_{a \in A} f(a, B)$
- Let $\delta(A, B) = \max\{g(A, B), g(B, A)\}$
- (X, δ) is Hausdorff's metric space

138. Hausdorff's Metric Topology

Is it compact?

- N/A

139. The Post Office Metric

Definition:

- Let (X, d) be the plane with the usual metric, and let 0 be the origin in the plane
- Define d^* on X by the formula $d^*(p, q) = d(0, p) + d(0, q)$ when $p \neq q$, and $d^*(p, q) = 0$ when $p = q$

139. The Post Office Metric

Is it compact?

- No

140. The Radial Metric

Definition:

- Let (X, d) be the plane with the usual metric, and let 0 be the origin in the plane
- We define d^* on X by:
- $d^*(p, q) = 0$ if $p = q$,
- $d^*(p, q) = d(p, q)$ if $p \neq q$ and the line through p and q passes through 0
- $d^*(p, q) = d(p, 0) + d(q, 0)$ otherwise

140. The Radial Metric

Is it compact?

- No

141. Radial Interval Topology

Definition:

- X is the plane, and the topology is generated by all open intervals disjoint from the origin which lie on lines passing through the origin, together with sets of the form $\bigcup\{I_\theta : 0 \leq \theta < \pi\}$, where each I_θ is a non-empty open interval centered at the origin on the line of slope $\tan\theta$

141. Radial Interval Topology

Is it compact?

- No

142. Bing's Discrete Extension Space

Definition:

- Let $\mathcal{P}$ denote the power set of $\mathbb{R}$, and let $X = \prod_{\lambda \in P} \{0, 1\}_\lambda$, where $\{0, 1\}_\lambda$ is a copy of the two point discrete space
- For each $r \in \mathbb{R}$, let x_r be the point of X whose λ'th coordinate equals 1 iff $r \in \lambda$
- Let $M = \{x_r \in X : r \in \mathbb{R}\}$
- X has the product topology, and $X \setminus M$ is dense in X, so we can form the discrete extension of X by $X \setminus M$

142. Bing's Discrete Extension Space

Is it compact?

- No

143. Michael's Closed Subspace

Definition:

- Let Y be the subspace $M \cup F$ of Bing's discrete extension space, where F is the collection of all finite sets in $X \setminus M$

143. Michael's Closed Subspace

Is it compact?

- No

Section 3

References

References I

- Counterexamples In Topology by Lynn Steen and J. Arthur Seebach, Jr