Math 155B - Spring 2020 - "Mini"-Midterm 1 - April 16, 2020 - 12:45pm

Duration: 45 minutes

Instructions: Read completely before starting! You have 45 minutes once you start the midterm.

- Hand in your answers to the four questions as the answers to problems 1-4 on Gradescope.
- Hand in this cover page (or other statement of Academic Integrity along with the start and stop time) as the answer to problem 5 on Gradescope.
- You may (1) print out the quiz and write answers on the printed sheet, or (2) use a tablet to write on the PDF file, or (3) write answers on a blank sheet of paper (preferably white, unlined printer paper).
- **BEFORE YOU START WORKING OR THINKING ABOUT THE PROBLEMS:** Write the start time in the space below.
- **WHEN YOU STOP:** Write the stop time in the space below. The total time should be at most 45 minutes. If more than 50 minutes, explain in the comments below.
- **AFTER YOU STOP:** Sign the Academic Integrity Acknowledgement below.
- Convert your written answers to a PDF file.
- **UPLOAD TO GRADESCOPE – IMMEDIATELY AFTER THE STOP TIME:** If there are problems uploading, please explain in the comment section. If you modify any answers after the "STOP TIME", that is, while preparing to upload, please explain in the comments below.

Academic Integrity Guidelines: **You must work this exam on your own. You may use the supplied "cheat sheet", but may not use any other notes, textbook, online resources, or resources of any kind. You may neither receive help nor provide help on this quiz.**

START TIME:

STOP TIME:

ACADEMIC INTEGRITY: I understood and abided by the academic integrity guidelines.

SIGNED: ________________________

Comments (optional):

Math 155B - Spring 2020 - Quiz #2 - April 14, 2020 - 1:00pm

1. (Convexity and Variation Diminishing)

a. Draw and label a picture of a convex set and a picture of a non-convex set.

b. Suppose a degree k Bézier curve has control points $\mathbf{p}_0, \ldots, \mathbf{p}_k$. What does the *convex hull property* tell you about the Bézier curve?

c. What does the *variation diminishing property* tell you about the Bézier curve?

d. Suppose $\mathbf{q}(u)$ (for $0 \leq u \leq 1$) is a degree three Bézier curve in $\mathbb{R}^2$, and that L is a line. What is the maximum number of times that $\mathbf{q}(u)$ can cross L? Justify your answer.

2. These questions concern a shader program with only a vertex shader and a fragment shader, which do not access any texture or buffer, running with the usual behaviour of the OpenGL pipeline. It is also assumed the vertex and fragment shader do not know the details of the overall geometry being rendered. If more than one answer is possible, please explain.

a. Can a vertex shader know what the depth of a vertex will be? In particular, can the vertex shader determine if the vertex will pass the near-clipping plane test?

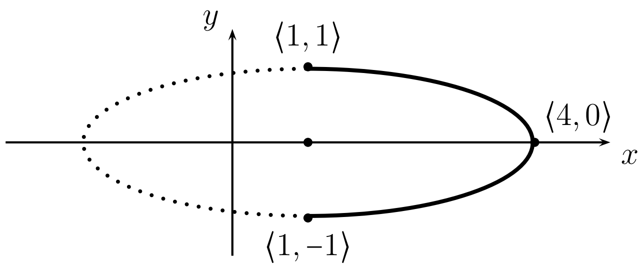
b. Can a vertex shader tell whether the vertex appears in both a front-facing triangle and a back-facing triangle?

c. When rendering a pixel from a triangle, can the fragment shader know the Vertex ID's of the three vertices that form the triangle?

d. When rendering a pixel from a triangle, can the fragment shader tell whether the triangle is front-facing?

e. Explain what the "Vertex ID" of a vertex is.

3. This question concerns a rational Bézier curve in $\mathbb{R}^2$.
An ellipse in $\mathbb{R}^2$ is centered at $\langle 1, 0\rangle$ and has major radius 3 and minor radius 1. Its major radius is along the x-axis; its minor radius is parallel to the y-axis. Thus it goes through the four points $\langle 1, \pm 1\rangle$, $\langle -2, 0\rangle$ and $\langle 4, 0\rangle$.



a. Express the right half of this ellipse as a degree 2 Bézier curve by giving its control points.

b. Now express the same curve as a degree 3 Bézier curve.

4. A particle is moving in $\mathbb{R}^2$ and its position at time u is given by a degree 3 polynomial function $\mathbf{q}(u)$. Its position at time $u = 0$ is $\mathbf{q}(0) = \langle 0, 0 \rangle$. Its position at time $u = 1$ is $\mathbf{q}(1) = \langle 3, 0 \rangle$. Its velocity at time $u = 0$ is $\mathbf{q}'(0) = \langle 0, 2 \rangle$. Its velocity at time $u = 1$ is $\mathbf{q}''(1) = \langle 0, 0 \rangle$.

a. Express $\mathbf{q}(u)$ as a degree 3 Bézier curve by giving its control points.

b. Express the particle's velocity (the derivative $\mathbf{q}'(u)$) as a degree 2 Bézier curve by giving its control points.

c. Express particle's acceleration (the second derivative $\mathbf{q}''(u)$) as a degree 1 Bézier curve by giving its control points.