Math 155B — Computer Graphics — Spring 2020
Homework #4 — Due Tuesday, April 14, 9:00pm
Hand in via Gradescope

1. Let $\mathbf{q}(u)$ be the rational, degree two Bézier curve with homogeneous control points $\mathbf{p}_0 = \langle 1, 0, 1 \rangle$, $\mathbf{p}_1 = \langle \sqrt{2}/2, \sqrt{2}/2, \sqrt{2}/2 \rangle$ and $\mathbf{p}_2 = \langle 0, 1, 1 \rangle$. Prove that this Bézier curve traces out the 90° arc of the unit circle in $\mathbb{R}^2$ from the point $\langle 1, 0 \rangle$ to $\langle 0, 1 \rangle$. See Figure VIII.19 in the book for this.

2. Apply degree elevation to the degree two Bézier curve of the previous problem to obtain a degree three Bézier curve that traces out the same portion of the circle.

3. The degree two Bézier curve with control points $\mathbf{p}_0 = \langle 0, 1, 1 \rangle$ and $\mathbf{p}_1 = \langle 1, 0, 0 \rangle$ and $\mathbf{p}_2 = \langle 0, -1, 1 \rangle$ traces out the right half of the unit circle. Apply recursive subdivision to this curve to obtain a degree two curve that traces out the first quadrant of the unit circle centered at the origin. ***(Control points are corrected from first posting.)***

4. The degree two Bézier curves of problems 1 and 3 both trace out the same quadrant of the unit circle. However, they have different control points. Why is this not a contradiction? In what way are the two Bézier curves different?

5. Prove that, for a given degree k Bézier curve, there is a *unique* set of control points $\mathbf{p}_0, \ldots, \mathbf{p}_k$ which defines that Bézier curve. I.e., two different sequences of $k + 1$ control points define two different Bézier curves. [Hint: This should be clear for $\mathbf{p}_0$ and $\mathbf{p}_k$; for the rest of the control points, use induction on the degree and the formula for the derivative of a Bézier curve.]

6. Let $\mathbf{q}(u)$ be a degree k polynomial curve. Prove that there are control points $\mathbf{p}_0, \ldots, \mathbf{p}_k$ which represent $\mathbf{q}(u)$ as a degree k Bézier curve for $u \in [0, 1]$. [Hint: Prove that the dimension of the vector space of all degree k polynomial curves is equal to the dimension of the vector space of all degree k Bézier curves. You will need to use the previous exercise.]

7. Prove that there is no nonrational Bézier curve which traces out a nontrivial part of a circle. [Hint: A nonrational Bézier curve of degree k is of the form $\langle x(u), y(u) \rangle$ with $x(u)$ and $y(u)$ degree k polynomials. To have only points on the unit circle, they must satisfy $(x(u))^2 + (y(u))^2 = 1$. You might find it easier to think about the degree $k = 2$ case first.]

8. Give a full acknowledgement of assistance. This includes anyone or any web site, etc., that helped you; and anyone you helped.