Prop. \( \dim \text{Fl}(d, n) = \sum_{i=1}^{r} (d_i - d_{i-1})(n - d_i) \)

and \( \text{Fl}(d, n) \) is irreducible.

In particular, \( \dim \text{Fl}(n) = \binom{n}{2} \).

pf. Consider flag where \( W_d = \text{span}(e_1, \ldots, e_{d_1}) \),

\[
\text{stab}(W_d, \ldots, W_{d_r}) = \begin{pmatrix}
\vdots \\
0 \\
\vdots \\
d_2 - d_1 \\
d_1 \\
1
\end{pmatrix}
\]

\( \Rightarrow \dim \text{Fl}(d, n) = \# \text{0's} \)

\[= \sum_{i=1}^{r} (d_i - d_{i-1})(n - d_i) \]

Another fact: given an algebraic map \( \Pi: X \to Y \) between irreducible varieties, s.t. \( \dim \Pi^{-1}(y) = e \) \( \forall y \),

\( \Rightarrow \dim X = e + \dim Y \).
Note, if \( d' \) is a subsequence of \( d \), have forgetful map \( \text{Fl}(d'; n) \rightarrow \text{Fl}(d, n) \).

Fibers are identified w/ products of flag varieties can use to compute \( \dim \text{Fl}(d, n) \) by induction.

For \( \text{Fl}(n) \), stabilizer of any pt is a Borel subgroup. Also, every Borel subgroup is stabilizer of some pt.

\[
\left\{ \text{Borel subgroups} \right\} \overset{\text{of } \text{GL}_n}{\longrightarrow} \text{Fl}(n) \overset{\text{GL}_n/B}{\swarrow} \text{variety of Borel subgroups.}
\]

Any closed subgroup of \( \text{GL}_n(C) \) that contains a Borel subgroup is called parabolic.

Stabilizers of partial flags are parabolic, and they exhaust all of them.