Math 10B Final Exam Review Outline
Math 10B, Winter 2018

The final exam is cumulative, so you may be tested on any material from the entire course, but there will be a slight emphasis (approximately 40% of the problems) on the topics discussed in lecture during Weeks 8, 9, and 10 (sections 6.5, 7.1, 7.3, 7.4, 8.1, 8.3, 8.8). Below is a summary of the topics/skills you need to master from each section; in some cases, example problems from the textbook homework assignments are provided to illustrate what is meant by the particular skill.

- Section 4.8: Know how to find general antiderivatives (e.g. as in HW 4.8.15). Know how to find a specific antiderivative given a first or second derivative together with an initial condition (e.g. as in HW 4.8.25). Know how to solve word problems about linear motion (i.e. dropped or tossed objects) given information about acceleration and velocity.
- Section 5.1: Given a formula for a function f or a collection of input/output pairs of f in a table or on a graph, know how to use L_n, R_n, and M_n to approximate the area under the graph of f on a given interval. Given information about velocity, know how to use L_n, R_n, and M_n to approximate distance travelled (e.g. as in HW 5.1.12).
- Section 5.2: Know how to use L_n, R_n, and M_n to approximate definite integrals. Know that a definite integral is signed area and be able to use this together with the area formulas for circles, triangles, and rectangles to evaluate definite integrals (e.g. as in HW 5.2.31, 5.2.34, etc.). Know how to evaluate definite integrals using the properties:
  (1) If $b < a$, then $\displaystyle\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ (e.g. as in HW 5.2.40).
  (2) If $a < b < c$, then $\displaystyle\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$ (e.g. as in HW 5.2.48).
- Section 5.3: Know how to evaluate definite integrals using basic antiderivative formulas together with the FTC (e.g. as in HW 5.3.18). Know how to evaluate indefinite integrals using basic antiderivative formulas (e.g. as in HW 5.3.43). Know how to integrate the absolute value of a given function. Know how to calculate displacement and distance travelled (e.g. as in HW 5.3.59).
- Section 5.4: Know how to use the FTC to take the derivative of a function which is defined as an integral (e.g. as in HW 5.4.14). Given a graph of a function f, be able to understand the meaning of a function of the form $F(x) = \displaystyle\int_a^x f(t)\,dt$ in terms of signed areas, and be able to answer questions about the local extrema of F and identify the intervals on which F is increasing, decreasing, concave up, and concave down (e.g. as in HW 5.4.19).
- Section 5.5: Know how to evaluate indefinite integrals using u-substitution. Know how to evaluate definite integrals using u-substitution.

- Section 5.6: Know how to solve integrals using integration by parts. Remember that sometimes you need to apply integration by parts more than once and that sometimes you need to combine integration by parts with u-substitution.
- Section 5.7
  - Topic One: Know how to solve integrals using u-substitution together with the following identities:
    $$\sin^2(x) + \cos^2(x) = 1, \quad \sec^2(x) - \tan^2(x) = 1$$
    (e.g. as in HW 5.7.1 or HW 5.7.10).
  - Topic Two: Know how find the partial fraction expansion (PFE) of a given rational function and then use the PFE to find the antiderivative of the function.
  - Topic Three: Know how to solve integrals using trig substitution (know how to use the three substitutions $x = a\sin\theta$ and $x = a\tan\theta$ and $x = a\sec\theta$) (e.g. as in HW 5.7.16, but exam questions would not be multiple choice).
- Section 5.9: Know how to use the Trapezoidal Rule to approximate a definite integral.
- Section 5.10:
  - Topic One: Know how to use the definition of an improper integral as a limit to determine whether it converges or diverges. Know how to do both types of improper integrals: continuous function on infinite interval of integration (e.g. as in HW 5.10.8) and function with discontinuity at one endpoint of a finite interval of integration (e.g. as in HW 5.10.26). On the finite interval of integration problems, remember that you should write a one-sided limit, not a two-sided limit.
  - Topic Two: Know how to determine convergence or divergence of an improper integral using the Comparison Theorem. Note that if we expect you to use the Comparison Theorem to solve a problem, then we will tell you to use it in the instructions for that problem.
- Section 6.1: Know how to find the area of a plane region by setting up and a solving a definite integral. Note that as in your homework, if the region is enclosed by simple functions like lines and parabolas, you may be asked to sketch the region (e.g. as in HW 6.1.8 or HW 6.1.9).
- Section 6.2: Know how to find the volume of a solid by setting up and solving a definite integral. In particular, you may be asked to find the volume of a solid obtained by revolving a plane region about a line (e.g. as in HW 6.2.1 or HW 6.2.12), or you may be asked to find the volume of a solid described in terms of its base and cross-sections (e.g. as in HW 6.2.43).
- Section 6.5: Know how to find the average value of a given function on a given interval.
- Section 7.1: Know how to check if a specific function is a solution of a given differential equation.

- Section 7.3: Know how to use separation of variables to find the general solution of a differential equation (e.g. as in HW 7.3.3). Know how to use separation of variables to find the particular solution of a differential equation that satisfies a given initial condition (e.g. as in HW 7.3.13).
- Section 7.4: Know how to solve word problems that involve solving a differential equation. If there is a problem from this section on the exam, it will either be a Newton's Law of Heating/Cooling Problem (e.g. as in HW 7.4.13) or a problem where some population grows according to a differential equation of the form $dy/dt = ky$ (e.g. as in HW 7.4.1).
- Section 8.1: Know what a sequence is and make sure you understand sequence notation. Know how to determine whether a given sequence converges or diverges. If a sequence converges, know how to find its limit.
- Section 8.3: Given a geometric series, know how to find the first term and the common ratio. Know how to determine whether a given geometric series converges or diverges. If a geometric series converges, know how to find its sum. Note that you should be prepared to answer questions about geometric series written in longhand form (e.g. as in HW 8.2.11) or in shorthand (summation) notation (e.g. as in HW 8.2.15).
- Section 8.8: Given a function f and specific numbers a and n, know how to find the Taylor polynomial $T_n(x)$ for the function f with center a (e.g. as in HW 8.8.4).

In addition to the calculus topics, make sure to review your pre-calculus skills:

- Know how to evaluate trig functions at standard angles, e.g. $\sin(\pi/3)$, $\cos(\pi)$, etc.
- Know the definitions of $\sec(\theta)$, $\csc(\theta)$, $\arcsin(t)$, $\arctan(t)$
- Know how to graph basic functions:
  (1) $y = mx + b$, e.g. $y = 3x - 2$, $y = 6$, etc.
  (2) $y = ax^2 + bx + c$, e.g. $y = 12x^2 - 17x + 6$, etc.
  (3) $y = x^3$
  (4) $y = \sin x$
  (5) $y = \cos x$
  (6) $y = e^x$
  (7) $y = \ln x$