Math 10B | **Final Examination** | **Name:** ____________________
March 15, 2016 | **Version A** | **PID:** ____________________

Instructions

1. **Write your *Name*, *PID*, *Section*, and *Exam Version* on the front of your Blue Book.**
2. **No calculators or other electronic devices are allowed during this exam.**
3. **You may use one page of notes, but no books or other assistance during this exam.**
4. **Read each question carefully, and answer each question completely.**
5. **Write your solutions clearly in your Blue Book.**
   (a) **Carefully indicate the number and letter of each question and question part.**
   (b) **Present your answers in the same order as they appear in the exam.**
   (c) **Start each numbered problem on a new side of a page.**
6. **Show all of your work. No credit will be given for unsupported answers, even if correct.**
7. **Write Name & PID on this exam sheet and return inside front cover of your Blue Book.**

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0. (2 points) Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

1. (6 points) Evaluate the indefinite integral $\displaystyle\int \frac{1}{x^2+4}\,dx$. [Hint: Use $x = 2\tan(\theta)$.]

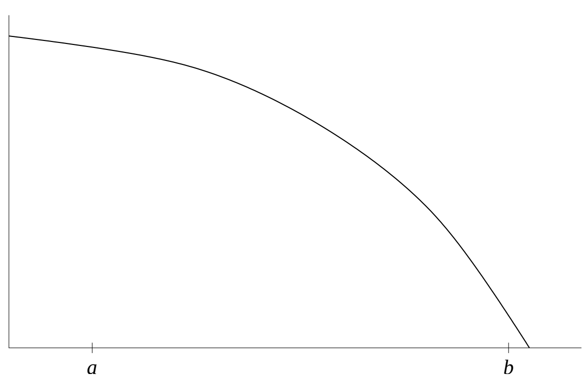
2. (6 points) What is the volume of the solid obtained by revolving the region bounded by $x = 0$, $x = 2$, and $\dfrac{x}{\sqrt{x^3+2}}$ about the x-axis?

3. (6 points) Let $f(x) = \dfrac{1}{2+x}$.
   (a) Find the third degree Taylor polynomial for $f(x)$ centered at $a = 0$.
   (b) Find the third degree Taylor polynomial for $f(x)$ centered at $a = 1$.

4. (6 points) The graph of a function f is given below. Put the following approximations to the integral $\int_a^b f(x)\,dx$ and its exact value in order from smallest to largest:

   $LEFT(n)$, $RIGHT(n)$, $MID(n)$, $TRAP(n)$, $EXACT\ VALUE$.

   Briefly explain how you arrived at your answer based on whether f is increasing or decreasing and whether its graph is concave up or concave down.



Note: Problems 5 – 8 are on the other side of this page.

(This exam is worth 50 points.)

5. (6 points) The repeating decimal number $q = 0.81818181\ldots$ can be written in the form

$$q = 81\left(\frac{1}{100}\right) + 81\left(\frac{1}{100}\right)^2 + 81\left(\frac{1}{100}\right)^3 + 81\left(\frac{1}{100}\right)^4 + \cdots.$$

Write q as a rational number $\dfrac{m}{n}$, where m and n are integers with no common factors.
[Hint: What does the geometric series representing q converge to?]

6. (6 points) A population's growth is described by the differential equation

$$\frac{dP}{dt} = kP(L-P),$$

where k and L are constants. (L is often called the "carrying capacity" of the population.)

(a) Find the general solution to the differential equation.
You may use the fact that $\frac{1}{P(L-P)} = \frac{1}{L}\left(\frac{1}{P} + \frac{1}{L-P}\right)$.

(b) What are the equilibrium (constant) solutions? Explain how you determined them.

7. (6 points) Using the table below, estimate $\displaystyle\int_0^{10} f(t)\,dt$ using left–hand, right–hand, and trapezoidal estimates.

t	0	5	10
$f(t)$	1.2	2.8	4.0

8. (6 points) Use the **Comparison Test** to determine whether the following improper integral converges or diverges.

$$\int_1^{\infty} \frac{\cos(x) + 8}{\sqrt{x^7 + 10}}\,dx$$