Math 121A, HW3

Please note that this homework assignment is two pages long.

**Notes on Problem One:** In Problem One, you will test some of the population models that your classmates or students from previous classes proposed for the Population Problem. When you are asked to “find and interpret the percent error”, it means that you need to make a calculation and then interpret your answer in words, e.g. “The predicted population overestimates the actual population by three percent”.

(1) (a) Some students tried to solve the Population Problem using linear models:
   (i) One proposed linear model was
   \[ P(x) = 109.38 + 3.67(x - 2017) \]
   where \( x \) is the year. Use this model to find the population in 2007, and compare it to the actual population in 2007. Find and interpret the percent error. Next use this model to find the population in 1998, and compare it to the actual population in 1998. Find and interpret the percent error.
   (ii) Another proposed linear model was
   \[ P(x) = 2.20863x + 63.714 \]
   where \( x \) is the number of years after 1997. Use this model to find the population in 2007, and compare it to the actual population in 2007. Find and interpret the percent error. Next use this model to find the population in 2017, and compare it to the actual population in 2017. Find and interpret the percent error.

(b) Some students tried exponential models:
   (i) One proposed exponential exponential was
   \[ P(t) = 109.38(1.025)^t \]
   where \( t \) is the number of years after 2017. Use this model to find the population in 2007, and compare it to the actual population in 2007. Find and interpret the percent error. Next use this model to find the population in 1998, and compare it to the actual population in 1998. Find and interpret the percent error.
   (ii) Another proposed exponential model was
   \[ P(t) = 67.38e^{rt} \]
   where \( t \) is the number of years after 1998, and \( r = \frac{\ln \left( \frac{109.38}{67.38} \right)}{19} \). Use this model to find the population in 2007, and compare it to the actual population in 2007. Find and interpret the percent error. Next use this model to find the population in 2017, and compare it to the actual population in 2017. Find and interpret the percent error.

(c) Some students proposed the quadratic model
   \[ P(n) = 67.38 + 1.75n + .05n(n - 1) \]
   where \( n \) is the number of years after 1998. Use this model to find the population in 2007, and compare it to the actual population in 2007. Find and interpret the percent error. Next use this model to find the population in 2017, and compare it to the actual population in 2017. Find and interpret the percent error.
(2) Many of my students in the past have used the following approach to the Population Problem. After discovering that the sequence of second differences is basically constant, they assume that the population is given by some function of the form \( P(t) = At^2 + Bt + C \), where \( t \) is the number of years after 1998. Carry out the details of this approach to find \( A, B, \) and \( C \); namely, choose three input/output pairs from the given data to set up a system of three linear equations in \( A, B, \) and \( C \), and solve the system (feel free to solve the system online using Wolfram Alpha; you do not have to show the calculations). Is your population function the same as or different from Lois’ proposed model from Problem One part (c)?

(3) You are teaching a Math 121A class, and you find two of your students having a heated discussion. Student A asserts that the answer to the problem that they are working on must be an exponential function of the form \( y = ab^t \), where \( a \) and \( b \) are some non-zero numbers with \( b > 0 \). Instead, Student B says that his high school teacher taught him that exponential functions are always of the form \( y = Pe^{rt} \), where \( P \) and \( r \) are some fixed non-zero numbers. Help them to resolve their discussion by either (1) Proving that any function of the form \( y = ab^t \) can be written in the form \( y = Pe^{rt} \) and vice versa or (2) Constructing a specific example of a function \( y = ab^t \) that cannot be written in the form \( y = Pe^{rt} \), or vice versa.

(4) Prove that \( \{a_n\}_{n \geq 1} \) is a linear (aka arithmetic) sequence if and only if its sequence of first differences is a constant (non-zero) sequence.

(5) Prove that \( \{a_n\}_{n \geq 1} \) is an exponential (aka geometric) sequence if and only if the ratio of consecutive terms is a non-zero constant.