Math 121A: The Method Behind the Madness

Laura J. Stevens

Department of Mathematics, UCSD

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General Goals of the 121 Series:

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- Advancing your knowledge of mathematics

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- Advancing your knowledge of student learning (i.e. your view and understanding of the process of learning mathematics)

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- Advancing your knowledge of pedagogy (i.e. teaching practices)

DNR-Based Instruction in Mathematics

A Research-Based Framework for Teaching Mathematics: **DNR-Based Instruction in Mathematics**

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To learn more about DNR: http://www.math.ucsd.edu/~harel

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- Mental acts are basic elements of human cognition. To describe, analyze, and communicate about humans intellectual activities, one must attend to their mental acts.

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Example One

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- Mathematical symbols can have multiple interpretations (would be manifested by one who exhibits more than one of the ways of understanding).
- It is advantageous to attribute different interpretations to a mathematical symbol in the process of solving problems (would be manifested by one who can vary the interpretation of the symbols according to the problem at hand).

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- Look for a simpler problem
- Consider alternative possibilities while attempting to solve the problem
- Just look for key words in the problem statement

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- Authoritative proof scheme: because the teacher says it's true
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- Deductive proof scheme: one proves an assertion through a finite sequence of steps which follows from premises (and previous conclusions) through the application of rules of inference

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Target Instructional Objectives

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PGA way of thinking:

The ability to fluently connect the physical/perceptual aspects of a problem situation with the geometric aspects (e.g. graph) and the algebraic aspects (e.g. formulas and equations). One who possesses the PGA way of thinking searches for and exploits the correspondences between the physical, geometric, and algebraic aspects of a mathematical topic.

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  - algebraic proof
  - geometric proof
- Let $g(x) = |x|$.
  1. Sketch the graph of the function g and explain why the graph of g suggests that $\frac{dg}{dx}(0)$ does not exist.
  2. Use the $\varepsilon - \delta$ definition of the derivative to prove that $\frac{dg}{dx}(0)$ does not exist.

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- Let $f(x) = 4x^2 + 2x$.
  1. Sketch the graph of f. Sketch the tangent line to the graph of f at the point $(-1, f(-1))$.
  2. The slope of the tangent line to the graph of f at the point $(-1, f(-1))$ is -6. This means that on a small interval around -1, the slopes of the secant lines through $(-1, f(-1))$ and $(-1 + \Delta x, f(-1 + \Delta x))$ should be close to -6. How small does $|\Delta x|$ need to be to guarantee that $\frac{\Delta f}{\Delta x}(-1, \Delta x)$ is within 0.01 of -6?

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- Let $g(x) = x^2 - 6x + 7$. Verify that the conclusion of the Mean Value Theorem is true for the function g on the interval $[2, 8]$. Illustrate your answer with a sketch to demonstrate what is happening geometrically.

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Thinking in Terms of Functions as Processes and Models of Reality:

One who possesses this way of thinking understands a function as a dynamic transformation of quantities according to some repeatable means which, given the same original quantity, will always produce the same transformed quantity. In contrast, the most elementary conception of functions involves the ability to plug into an algebraic expression and calculate.

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- Joe and Kamala go for a run along the same route. The route starts with a five kilometer uphill run along a straight path, which is followed by a five kilometer downhill run back to the starting point along the same straight path as the uphill portion. Joe begins his run 10 minutes before Kamala and runs at the of 12 km/hr uphill and the rate of 18 km/hr downhill. Kamala runs at the rate of 15 km/hr uphill and at the rate of 20 km/hr downhill.
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Deductive proof scheme:

The ability to produce deductive proofs and, in particular, the ability to conjecture, apply mental operations that are goal oriented, and understand that all justification must be ultimately based on inference rules.

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- Prove Rolle's Theorem.
- Prove that if two functions have the same derivative, then the functions differ by a constant.

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Definitional Reasoning:

A way of thinking by which one defines objects and proves assertions in terms of mathematical definitions. A mathematical definition is a description that applies to all objects to be defined and only to them. A crucial feature of this way of thinking is that, with it, one is compelled to conclude logically that there can be only one mathematical definition for a concept within a given theory; namely, if D_1 and D_2 are such definitions for a concept C, then D_1 is a logical consequence of D_2, and vice versa; otherwise, C is not well defined.

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- Prove that if $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function and $f'(x_0) > 0$, then there exists some number $r > 0$ so that if $x \in (x_0, x_0 + r)$, then $f(x) > f(x_0)$, and if $x \in (x_0 - r, x_0)$, then $f(x) < f(x_0)$.

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- Prove the FTC II (proof uses the algebraic definition of the definite integral as a limit of a Riemann sum).

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- In 1997, the California State Board of Education adopted its own state "Mathematics Content Standards".

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  - uneven, i.e. no difficult content in one grade and too much difficult content in another

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- Currently 41 states employ the Common Core State Standards in Mathematics.

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  8. Look for and express regularity in repeated reasoning.

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- Structure lessons to allow repeated reasoning about concepts and ideas, to allow for internalization – a conceptual state where one is able to apply knowledge autonomously and spontaneously – and organization of knowledge.

Exportable Teaching Practices for Your Consideration

- Realize instructional goals in terms of both ways of understanding (such as definitions, theorems, proofs, problems and their solutions, and so on) and ways of thinking, and pay attention to the developmental interdependency between these two categories of knowledge.
- Structure lessons to allow repeated reasoning about concepts and ideas, to allow for internalization – a conceptual state where one is able to apply knowledge autonomously and spontaneously – and organization of knowledge.
- Expect the process of learning to often involve confusion, and adjust the trajectory of learning based on estimations of the learners' background knowledge.

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- Recognize and take steps to help surmount the difficulty involved in conceptualizing particular mathematical concepts and ideas, e.g. the difficulty of conceptualizing fraction as a number, of transitioning from empirical reasoning to deductive reasoning, etc. Try to minimize and help students get past didactical obstacles (those that are the result of narrow instruction).

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- Emphasize meaning, encourage creativity, and give your students every opportunity to articulate mathematics. Encourage your students to **seek causality**, to always ask why! Allow them to experience mathematics for what it really is.

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