Math 20B Midterm Two Review Outline, Winter 2020

The emphasis of the Math 20B second midterm exam will be textbook Sections 7.2, 7.3, 7.5, 7.7, 10.1, 10.2, and Supplement Sections 1, 2, 3, and 5. (We will not ask you any questions related to Supplement Section 4). Please do note that since math builds on itself, problems from these sections also require knowledge from earlier in the course such as u-sub, integration by parts, etc.

Below is a summary of the topics/skills you need to master from each of the aforementioned sections. We suggest that you start by solving the examples from your lecture notes, and then moving on to re-doing enough of the assigned homework problems from each section in order to feel confident about the main concepts. When you are solving the aforementioned problems, it is of the utmost importance that you attempt to solve them without looking at our solutions or solutions/hints on Webassign. This is the only way to really make the solutions your own. If you are able to understand and solve all of the lecture examples and the homework problems on your own and without any assistance, then you should do very well on this exam.

- **Supplement 1:** Know how to write a complex number $a + bi$ in the polar form $r(\cos \theta + i \sin \theta)$, and vice versa. Know how to find the $n^{th}$ power of a given complex number using DeMoivre’s Theorem. Know how to find all of the $n^{th}$ roots of a given complex number.

- **Supplement 2:** Know how to write a complex number $a + bi$ in the form $re^{i\theta}$, and vice versa. Know how to integrate a complex exponential function, e.g. $\int e^{2ix} \, dx$.

- **Supplement 3:** Know how to solve integrals such as $\int \cos(2x) \sin(5x) \, dx$ or $\int e^{7ix} \cos(4x) \, dx$ using the formulas $\cos x = \frac{e^{ix} + e^{-ix}}{2}$ and $\sin x = \frac{e^{ix} - e^{-ix}}{2i}$.

- **Section 7.2:** Know how to solve integrals such as $\int \sin^3(x) \cos^4(x) \, dx$ and $\int \sec^4(x) \tan^5(x) \, dx$ using a combination of a u-sub and the trig identities: $\sin^2 x + \cos^2 x = 1$ or $\sec^2 x - \tan^2 x = 1$.

- **Section 7.3:** Know how to solve integrals using trig substitutions. There are three types of trig subs that you can use, so you need to be able to pick the correct one: $x = a \sin \theta$ or $x = a \tan \theta$ or $x = a \sec \theta$. Remember that at the end of most of these problems, you need to use your little triangle to write your final antiderivative in terms of $x$.

- **Section 7.5 and Supplement 5:** Know how to find the partial fraction expansion (PFE) of a given rational function and then use the PFE to integrate the function. Remember that if the degree of the numerator is equal to or greater than the degree of the denominator, then you must do long division before the PFE.

- **Section 7.7:**
  - Know how to use the definition of an improper integral as a limit to determine whether it converges or diverges. Know how to handle both types of improper integrals: continuous function but infinite interval of integration OR finite interval of integration but integrand is discontinuous at one of the endpoints. On the finite interval of integration problems, you should write a one-sided limit, not a two-sided limit.
  - Know how to determine convergence or divergence using the Comparison Test for Improper Integrals.

- **Section 10.1:**
  - Know how to write out the first few terms of a sequence.
  - Know how to determine if a given sequence converges or diverges (often we need to use Math 20A techniques such as L’Hopital’s Rule).
  - If a sequence converges, know how to find its limit.

- **Section 10.2:**
  - Know how to find the first few partial sums of a given series.
  - Given a geometric series in shorthand summation notation $\sum ra^n$ or in longhand notation $a + ar + ar^2 + ar^3 + \cdots$, be able to find its common ratio and identify if it converges or diverges.
  - Be able to find the sum of a convergent geometric series.
  - Be able to use a partial fraction expansion to find a formula for the $N^{th}$ partial sum of a telescoping series and then find the sum of the series (e.g. like problem 4 in 10.2 online homework).
  - Remember: If $\lim a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.