Instructions
1. Write your Name and PID in the spaces provided above.
2. Make sure your Name is on every page.
3. No calculators, tablets, phones, or other electronic devices are allowed during this exam.
4. Put away ANY devices that can be used for communication or can access the Internet.
5. You may use one handwritten page of notes, but no books or other assistance during this exam.
6. Read each question carefully and answer each question completely.
7. Write your solutions clearly in the spaces provided. Work on scratch paper will not be graded.
8. Show all of your work. No credit will be given for unsupported answers, even if correct.

(1 point) 0. Carefully read and complete the instructions at the top of this exam sheet and any additional instructions written on the chalkboard during the exam.

(6 points) 1. Compute \( \int \cos^{-1}(y) \, dy \). (Hint: Recall that \( \frac{d}{dy} \cos^{-1}(y) = -\frac{1}{\sqrt{1-y^2}} \))

\[
\begin{align*}
\int u_1 = \cos^{-1}(y) & \, dy \\
\frac{du_1}{dy} &= -\frac{1}{\sqrt{1-y^2}} \\
v_1 &= y \\
\int (\cos^{-1}(y) \, dy) &= y \cos^{-1}(y) - \int \frac{-y}{\sqrt{1-y^2}} \, dy \\
&= y \cos^{-1}(y) + \int \frac{y}{\sqrt{1-y^2}} \, dy \\
\int u_2 &= 1-y^2 \\
\frac{du_2}{dy} &= -2y \\
\frac{1}{2} \int du_2 &= y \, dy \\
&= y \cos^{-1}(y) + \frac{1}{2} \int \frac{1}{\sqrt{1-y^2}} \, dy \\
&= y \cos^{-1}(y) - \sqrt{1-y^2} + C.
\end{align*}
\]
2. Compute the following integrals.

(a) \[ \int_1^e \frac{\ln(y)}{y} \, dy \]

Let \( u = \ln(y) \)

\[ u = \ln(1) = 0 \quad \text{at} \quad y = 1 \]

\[ u = \ln(e) = 1 \quad \text{at} \quad y = e \]

\[ \int_1^e \frac{\ln(y)}{y} \, dy = \int_0^1 u \, du = \frac{1}{2} u^2 \Bigg|_0^1 = \frac{1}{2} - 0 = \frac{1}{2} \]

(b) \[ \int_0^{\pi} y^3 \sin(y^2) \, dy \]

Note \( \int_0^{\pi} y^3 \sin(y^2) \, dy = \int_0^{\pi} y^2 \sin(y^2) \, dy \)

Let \( w = y^2 \)

\[ dw = 2y \, dy \]

\[ \frac{1}{2} \, dw = y \, dy \]

\[ y = \sqrt{\pi} \quad w = (\sqrt{\pi})^2 = \pi \]

Then \( \int_0^{\pi} y^3 \sin(y^2) \, dy = \int_0^{\pi} w \sin(w) \left( \frac{1}{2} \right) \, dw \)

\[ = \frac{1}{2} \int_0^{\pi} w \sin(w) \, dw \]

\[ = \frac{1}{2} \left[ -w \cos w \Bigg|_0^{\pi} + \int_0^{\pi} \cos(w) \, dw \right] \]

\[ = \frac{1}{2} \left[ -\pi \cos(\pi) - 0 \right] \]

\[ = \frac{1}{2} \left[ \pi + 0 \right] = \frac{\pi}{2} \]
(6 points) 3. Compute the volume of a truncated pyramid with square base of side length 3, height 2, and whose top is a square of side length 1.

Base is a square.

Area of square = $(3-z)^2$

$$V = \int_0^2 (3-z)^2 \, dz = -\frac{1}{3} (3-z)^3 \bigg|_0^2 = -\frac{1}{3} + \frac{1}{3} (3)^3 = \frac{26}{3}$$
4. Find the area of the region inside one petal of the curve $r = 4 \sin(3\theta)$, but outside the circle $r = 2$.

[Hint: You might find the identity $\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$ useful for evaluating the integral.]

First find points of intersection.

Solve $4\sin(3\theta) = 2 \Rightarrow \sin(3\theta) = \frac{1}{2}$.

Consider $\sin(x) = \frac{1}{2} \Rightarrow x = \frac{\pi}{6} + 2\pi k$

Say $3\theta = \frac{\pi}{6}$ or $3\theta = \frac{5\pi}{6}$

Then $A = \frac{1}{2} \left[ \int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} 16\sin^2(3\theta) - 4 \, d\theta \right]$

$= \frac{1}{2} \left[ \int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} 16\left(\frac{1}{2} - \frac{1}{2}\cos(6\theta)\right) - 4 \, d\theta \right]$}

$= \frac{1}{2} \left[ \int_{\frac{\pi}{18}}^{\frac{5\pi}{18}} -8\cos(6\theta) \, d\theta \right] = \frac{1}{2} \left[ 4\theta \right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}} - \frac{8}{6} \sin(6\theta) \right]_{\frac{\pi}{18}}^{\frac{5\pi}{18}}$

$= \frac{1}{2} \left[ \frac{16\pi}{18} - \frac{4}{3} \left( \sin\left(\frac{5\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right) \right) \right]$

$= \frac{1}{2} \left[ \frac{8\pi}{9} - \frac{4}{3} \left( -\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right) \right] = \frac{1}{2} \left[ \frac{8\pi}{9} + \frac{4\sqrt{3}}{3} \right] = \frac{8\pi}{9} + \frac{2\sqrt{3}}{3}$