Your Thirty-five Minute\textsuperscript{1} Assignment:

(1) The so-called “Gauss Summation Formula” states that for any positive integer $k$, we have:
\[1 + 2 + 3 \cdots + k = \frac{k(k + 1)}{2}\]

Last week, some students used this formula in their solutions to the Stair-like Structure Problem. \textbf{Your job:} Spend 20 minutes attempting to prove the Gauss Summation Formula. Please be ready to present your ideas, partial or complete, in seminar tomorrow.

(2) We will also continue to see solutions to the Stair-like Structure Problem. Spend 15 minutes reviewing and preparing to present your ideas about the Stair-like Structure Problem in seminar tomorrow.

\textbf{Stair-like Structure Problem:} A figure such as the one below is called a stair-like structure.

\begin{center}
\begin{tikzpicture}
\draw (0,0) -- (4,0) -- (4,4) -- (0,4) -- cycle;
\draw (1,0) -- (1,1) -- (4,1) -- (4,4) -- (1,4) -- cycle;
\draw (2,0) -- (2,2) -- (4,2) -- (4,4) -- (2,4) -- cycle;
\draw (3,0) -- (3,3) -- (4,3) -- (4,4) -- (3,4) -- cycle;
\end{tikzpicture}
\end{center}

(a) You have 1176 identical square pieces. Can you use all the pieces to construct a stair-like structure?

(b) You want to build a stair-like structure out of toothpicks. Is it possible to use exactly 2628 identical toothpicks to create a stair-like structure?

Note: When constructing a structure using toothpicks, if two squares lie side by side, their common edge is only made of one toothpick. For example, in the picture above, the structure would be built with exactly 40 toothpicks.

\textsuperscript{1}Of course you may spend more time on this if you feel inspired! :-)