Math 281C Homework 4

Due: 5:00pm, April 29th

1. Let $X_1, \dots, X_n$ be i.i.d. from the Gamma distribution $\Gamma(\alpha, \gamma)$ with unknown α and γ, whose p.d.f. is

$$f(x) = \frac{1}{\Gamma(\alpha)\gamma^{\alpha}} x^{\alpha-1} e^{-x/\gamma} \mathbb{1}(x > 0).$$

   (i) Show that $\Gamma(\alpha, \gamma)$ belongs to an exponential family.

   (ii) Find a sufficient statistic for (α, γ).

2. Let $X_1, \dots, X_n$ be i.i.d. from $\Gamma(\alpha, \gamma)$.

   (i) For testing $H_0 : \alpha \le \alpha_0$ versus $H_1 : \alpha > \alpha_0$, and $H_0 : \alpha = \alpha_0$ versus $H_1 : \alpha \ne \alpha_0$, show that there exist UMP unbiased tests whose rejections are based on $W = \prod_{i=1}^n (X_i/\bar{X})$.

   (ii) For testing $H_0 : \gamma \le \gamma_0$ versus $H_1 : \gamma > \gamma_0$, show that a UMP unbiased test rejects H_0 when $\sum_{i=1}^n X_i > C(\prod_{i=1}^n X_i)$. Here, $C(t)$ is a function of t.

3. Let X and Y be independently distributed according to negative binomial distributions $Nb(p_1, m)$ and $Nb(p_2, n)$ respectively, and let $q_i = 1 - p_i$.

   (i) There exists a UMP unbiased test for testing $H_0 : p_1 \le p_2$ versus $H_0 : p_1 > p_2$.

   (ii) Determine the conditional distribution required for testing H_0 when $m = n = 1$.